

## MID-TERM BIOL 300: October 2007

**For all statistical tests, make sure that you clearly state your hypotheses. Unless otherwise stated, assume  $\alpha = 0.05$ . Show your work. Be as precise as possible about P-values.**

**Some questions have a box for the final answer. Please put the final answer in this box, and show all work in the other space provided, including the back of the page if necessary.**

**By taking this test and putting your name above, you are declaring that your answers on this test are all your own work.**

**Make sure that your copy of the test includes 8 pages, including this one.**

1. a. A team of researchers conducted hypothesis tests on 100 independent null hypotheses, using a significance level of  $\alpha = 0.05$ . They rejected none of them. If all the null hypotheses were true, what is the probability of getting the results that they got? (5 points)

1a. 0.0059

The probability of rejected a true null hypothesis is  $\alpha$ , or in this case 5%:

$$\Pr[0] = \binom{100}{0} (0.05)^0 (0.95)^{100} = 0.0059$$

1. b. A team of researchers conducted hypothesis tests on 100 independent null hypotheses, using a significance level of  $\alpha = 0.05$ . If all the null hypotheses were true, how many of these tests on average would be expected to reject the null hypothesis? (3 points)

If the null hypotheses are true and the tests use a significance level of 5%, the probability of a Type I error is 5%. Over 100 tests, this would give an average of 5 tests that (falsely) reject the null hypothesis.

1a. 5

2. 30% of the individuals in a very large population have a purple allele at one genetic locus, and the other 70% of individuals have an orange allele. If two alleles are chosen at random from this population, what is the chance that the two alleles are the same as each other? (4 points)

2. 0.58

$$\Pr[\text{first is purple}]\Pr[\text{second is purple}] + \Pr[\text{first is orange}]\Pr[\text{second is orange}] =$$

$$0.3 \times 0.3 + 0.7 \times 0.7 = 0.58$$

3. a. Assume that a particular null hypothesis is false. If a researcher decides to use a smaller value of  $\alpha$ , does this increase, decrease, or not affect the power of the test? Why? (5 points)

It decreases the power, because lowering  $\alpha$  makes it harder to reject any null hypothesis, true or false.

3. b. Assume that a particular null hypothesis is true. If a researcher decides to use a smaller value of  $\alpha$ , does this increase, decrease, or not affect the power of the test? Why? (5 points)

Technically it still reduces the power, because the power of the test is defined by its ability to detect a given effect. We also accepted the answer that the power was not affected, given that the null hypothesis is true.

Power is *not* the probability that the test gives the correct result.

4. A researcher wants to use the Poisson distribution to model the number of leaves that fall off a tree per autumn day. Compare two situations: *Situation A* in which the weather is constant for every day in the study, and *Situation B* in which some days are windy and other days are calm. Which Situation (*A* or *B*) is would match the Poisson distribution better? Why? (5 points)

Situation A would match the Poisson distribution better, since the probability is more likely to be the same for all leaves, as assumed by the Poisson distribution.

5. For each of the following pairs, circle the larger value. If the two are equal, put an equals sign between the two.  $X$  is some random variable,  $Z$  is drawn from standard normal distribution,  $t$ ,  $F$ , and  $\chi^2$  have their usual meanings. (15 points)

a.  $\boxed{\Pr[X > 12]}$   $\Pr[X > 15]$

b.  $\Pr[Z < 1.5] = \Pr[Z > -1.5]$

c.  $\Pr[t_5 > 3.36]$   $\boxed{\Pr[F_{6,2} > 19.33]}$

d.  $\boxed{\Pr[\chi^2_1 > 3]}$  0.05

With one degree of freedom, 5% of the probability is beyond  $\chi^2 = 3.84$ . Given that  $\Pr[\chi^2 > 3]$  has more probability in the tail, the probability must be greater than 5%.

e.  $\boxed{\Pr[t_{10} < -4]}$  0.001

Similar to the part d.

6. Body temperature of healthy humans has a normal distribution with mean 98.3°F and variance 0.5373. What is the probability that a healthy human has a body temperature between 98°F and 99°F? (8 points)

6. 0.49

$$Z_{98} = \frac{98 - 98.3}{\sqrt{0.5373}} = -0.41 \quad Z_{99} = \frac{99 - 98.3}{\sqrt{0.5373}} = 0.95$$

$$\Pr[Z < -0.41] = 0.34 \quad \Pr[Z > 0.95] = 0.17$$

$$\Pr[Z < -0.41 \text{ or } Z > 0.95] = 0.34 + 0.17 = 0.51$$

$$1 - 0.51 = 0.49$$

7. In India, the antiinflammatory drug diclofenac is given to livestock, and sometimes these treated animals are later eaten by Oriental white-backed vultures. More than 95% of the vulture population has disappeared in the last three decades, and one possible cause is kidney failure caused by diclofenac exposure. The following ten numbers give the concentration of diclofenac in vulture kidneys, in  $\mu\text{g/g}$ . Using these numbers, calculate the summary statistics below.

0.060   0.064   0.077   0.091   0.114   0.179   0.186   0.233   0.450   0.451

- a. Mean (**3 points**) 0.1905
- b. Median (**3 points**) 0.1465
- c. Standard deviation (**3 points**) 0.1486
- d. Variance (**3 points**) 0.0221
- e. Range (**3 points**)  $0.451 - 0.060 = 0.391$
- f. Standard error of the mean (**5 points**)

$$SE_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{0.1486}{\sqrt{10}} = 0.047$$

8. A survey of Americans found that 449 out of 1213 were able to identify that Mercury was the planet closest to the sun. 728 out of 1213 were able to name Krypton as the planet of Superman's birth. Do an appropriate statistical test to determine whether Americans are more or less able to identify Superman's birth planet than they are to identify the real planet closest to the sun. Give the P-value as precisely as possible from the statistical tables, and interpret your result in words. (15 points)

This is comparing the proportions in two groups. In other words there are two categorical variables (right or wrong, Mercury question vs. Krypton question) that we want to ask whether are independent.

<i>Observed</i>	Krypton	Mercury	total
Correct	728	449	1177
Incorrect	485	764	1249
total	1213	1213	2426

$H_0$ : Americans identify Krypton and Mercury correctly with equal proportions.

$H_A$ : Americans identify Krypton and Mercury correctly with unequal proportions.

The expected value for Krypton/correct, under the null hypothesis is  $(1213/2426) \times (1177/2426) \times 2426 = 588.5$ . By subtraction from row and column totals, we find the rest of the expected values:

<i>Expected</i>	Krypton	Mercury	total
Correct	588.5	588.5	1177
Incorrect	624.5	624.5	1249
total	1213	1213	2426

$$\chi^2 = \frac{(728 - 588.5)^2}{588.5} + \frac{(449 - 588.5)^2}{588.5} + \frac{(485 - 624.5)^2}{624.5} + \frac{(764 - 624.5)^2}{624.5} = 98.7$$

$\chi^2$  is greater than the largest critical value given in the table for  $df = (2-1)(2-1) = 1$ , so  $P < 0.001$ . We reject the null hypothesis that Americans are able to identify the closest planet to the sun as often as they can identify Superman's home planet. American's are more likely to know about Krypton than Mercury.

9. *Heliconia bihai*, a tropical plant called lobster claw, has two different flower types (green or red-green) on islands in the Lesser Antilles, which are pollinated by either males or females of a local hummingbird. The hummingbirds differ strongly between males and females in terms of the length and curvature of their bills, so it becomes interesting to know whether the flowers also differ in their length and curvature. Below are data from five randomly chosen individuals of green type of the plant, with measures of the curvature. Also shown below are the curvature measurements for five randomly chosen members of the red-green type of plant. Are the mean curvatures different between the green and red-green types? (State your hypotheses, give the  $P$ -value as precisely as possible from the tables, and interpret the answer in words.) (15 points)

Green: 26, 30, 26, 37, 26

Red-green: 22, 23, 31, 28, 28

For this question we want a two-sample  $t$ -test, because we want to compare two groups to ask whether they have the same mean. The data are not paired, so paired  $t$ -test is inappropriate.

$H_0$ : The mean curvature of the green type was equal to the mean curvature of the red-green type.

$H_A$ : The mean curvature of the green type was not equal to the mean curvature of the red-green type.

$$\bar{Y}_G = 29 \quad \bar{Y}_{RG} = 26.4$$

$$s_G = 4.80 \quad s_{RG} = 3.78$$

$$s_p^2 = \frac{4(4.80)^2 + 4(3.78)^2}{4 + 4} = 18.66$$

$$SE_{\bar{Y}_G - \bar{Y}_{RG}} = \sqrt{\frac{18.66}{5} + \frac{18.66}{5}} = 2.73$$

$$t = \frac{29 - 26.4}{2.73} = 0.95$$

So we cannot reject the null hypothesis. There is insufficient evidence to say that the means are different.

{Note: these data are a random small sample from a much larger data set in the real paper. The larger data set did in fact detect a difference in the way predicted by the pollinator beak shapes.}