MID-TERM BIOL 300: October 2008

For all statistical tests, make sure that you clearly state your hypotheses. Unless otherwise stated, assume α = 0.05. Show your work. Be as precise as possible about P-values.

Some questions have a box for the final answer. Please put the final answer in this box, and show all work in the other space provided, including the back of the page if necessary.

By taking this test and putting your name above, you are declaring that your answers on this test are all your own work.

Make sure that your copy of the test includes 6 pages, including this one.

1. In order to reach its feeding area, a fish must cross a dangerous patch of water where there is a 10% chance of being eaten by a predator, for each independent crossing. What is the probability that a fish can survive the first 10 crossings? (*10 points*)

1.	0.35

The probability that it survives one cross is 1-Pr[dies per cross] = 1 - 0.1 = 0.9.

Each crossing is independent, so the probability of surviving ten times is 0.9^{10} = 0.35.

2. Define Type I error. What is the relationship between the Type I error rate and the significance level of a hypothesis test? (*6 points*)

Type I error is rejecting a true null hypothesis. The probability of a Type I error is equal to the significance level, or α.

3. For each of the following scenarios, **identify the best statistical test to use** and **state the null hypothesis**. (Please note, do not give the answer to the specific question, but simply state the best test to use and the null hypothesis for the scenario.) (*5 points each*)

a. Comparing the mean length of left claws to mean length of right claws in a sample of hermit crabs. (Each crab is measured for both claws.)

Paired *t*-test.

H_0: The mean lengths of left and right claws are the same.

b. Asking whether elephant dung appears randomly distributed over space.

χ^2 Goodness of fit test

H_0: The number of elephant dung per square meter follows a Poisson distribution.

c. Twenty bird territories are examined, and for each territory three male birds compete to gain the territory. The study asks whether the largest male is most likely to win these territorial battles.

Binomial test. (Goodness of fit test is also an acceptable answer. Contingency analysis is not, because the failures and the success for each territory are not independent, and so the individuals are not a random sample.)

H_0: The largest bird succeeds one-third of the time.

d. Comparing the proportions of students who get A's, for an experiment where one group is allowed to study and the other group is required to stay up all the night at a party.

χ^2 contingency analysis

H_0: The proportion of A's is the same in the study group as the party group.

e. Asking whether there is a relationship between which drug a patient is given and whether they survive for a month after treatment, with a very large sample.

χ^2 contingency analysis

H_0: Survival and drug treatment are independent.

f. Asking whether there is a relationship between which of two drugs a patient is given and whether they survive for a month after treatment, in a small sample where the expected number of patients who die on one of the drugs is 2.3.

Fisher's exact test (because the expected values of one of the cells are too low to use the χ^2 test.)

H_0: Survival and drug treatment are independent.

4. We are often happy to do favors for other people when they have a particular need. For example, we are more willing to let someone use a photocopier when they ask "Can I go in front of you, because I am in a rush?" than when they give no reason: "Can I go in front if you?"

Some researchers believe that simply giving a reason -- using the word "because" -- may be enough to trigger this giving behavior, even when the reason is not a very good one. An experiment was done by approaching 60 people who were about to use a photocopy machine (Langer *et al*. 1978). In 30 cases (call these the "request only group", the investigator asked, "May I use the Xerox machine?" For the request only group, 18 people allowed the investigator to go first. In the other group -- call this the "bad reason" group-- the investigator said, "May I use the Xerox machine, because I need to use the

Xerox machine?" Of the 30 people approached in this way, 28 allowed the investigator to go first.

Do an appropriate hypothesis test to ask whether the "bad reason" approach is better or worse than the "request only" approach. (*15 points*)

Here are the observed values and the expected values for a contingency table.

Observed	request only group	bad reason group	totals
Say yes	18	28	46
Say no	12	2	14
totals	30	30	60

Expected	request only group	bad reason group	totals
Say yes	23	23	46
Say no	7	7	14
totals	30	30	60

We'll do a χ^2 contingency analysis, to test the hypotheses:

H_0: People agreeing to the request was independent of whether a reason was given.

H_A: People agreeing to the request was not independent of whether a reason was given.

$$\chi^2 = \frac{(18-23)^2}{23} + \frac{28(-23)^2}{23} + \frac{(12-7)^2}{7} + \frac{(2-7)^2}{7} = 9.32$$

There is (2-1)(2-1)= 1 degree of freedom, so the critical value is $\chi^2_{(0.05).1} = 3.84$. We therefore reject the null hypothesis, and say that compliance depends on the nature of the request.

More specifically, $P < 0.005$.

5. In the wake of this economic crisis, gold is getting more valuable. A new business opens to sell gold, and they advertise that they have extraordinarily accurate scales. You take a piece of gold bullion that you know to weigh 1.000 ounce, and have them measure it on their scale 10 times. The answers are

1.001 1.001 1.000 1.002 0.999 0.998 1.000 0.997 0.998 0.998

a. Do a hypothesis test to ask whether the scale is accurate. (*15 points*)

We should do a one-sample *t*-test, with the hypotheses:

H_0: The mean value of the measurements is 1.000.

H_A: The mean value of the measurements is not 1.000.

The mean and standard deviation of these measurements are

$\overline{Y} = 0.9994 \qquad s = 0.001647$

Therefore the standard error of the mean is

$$SE_{\overline{Y}} = \frac{s}{\sqrt{n}} = 0.001647/\sqrt{10} = 0.0005207$$

and $t = \dfrac{\overline{Y} - \mu_0}{SE_{\overline{Y}}} = \dfrac{0.9994 - 1.000}{0.0005207} = -1.15$, with 10-1 = 9 degrees of freedom.

This value of *t* is not outside the critical value of 2.26, so we do not reject the null hypothesis. There is no strong evidence that the scale is unfair.

b. What is the median measurement of the weight of the gold bullion? (*4 points*)

5b. 0.9995

Putting the measurements in order, we get

0.997 0.998 0.998 0.998 0.999 1.000 1.000 1.001 1.001 1.002

There are an even number of data points, so we look at the middle two values: 0.999 and 1.000. The average of these two middle points is 0.9995.

6. Population growth per year is approximately normally distributed among countries, with mean of 1.38% and standard deviation equal to 1.20%.

a. About what fraction of countries have a positive (greater than 0) population growth rate? (*5 points*)

6a. 0.875

As population growth is approximately normally distributed, we can use the standard normal distribution to answer this. Calculate $Z = \dfrac{x-\mu}{\sigma} = \dfrac{0-1.38}{1.2} = -1.15$. The probability of getting a Z > -1.15 is what we are interested in, but the table doesn't give it. We find Pr[Z >-1.15] = 1 - Pr [Z<-1.15] = 1-Pr [Z> 1.15]= 0.875.

b. What fraction of countries have a growth rate between -1% and 2%? (*5 points*)

6b. 0.6746

Pr[-1 < growth rate < 2] = 1 - Pr[growth rate <-1] - Pr[growth rate >2].

Calculate the Z values for both cases: $Z = \frac{x-\mu}{\sigma} = \frac{-1-1.38}{1.2} = -1.98$ and $Z = \frac{2-\mu}{\sigma} = \frac{2-1.38}{1.2} = 0.52$. So we need to calculate $1-\Pr[Z<-1.98]-\Pr[Z>0.52] = 1-0.02385-0.30153 = 0.6746$

7. A random sample of 200 elephants has a mean trunk length of 1.5 meters. Trunk length is normally distributed, and 95% of the elephants in the sample have trunks between 1.0 and 2.0 meters. Using the information from this sample, what is the 95% confidence interval for the mean length of elephant trunks? (*10 points*)

Because trunk length is normally distributed, 95% of values should fall between the mean minus 1.96 standard deviations and the mean plus 1.96 standard deviations. So the standard deviation is approximately 0.5/1.96 = 0.255 in this sample. Therefore the standard error is approximately $SE_{\bar{Y}} = \frac{0.255}{\sqrt{200}} = 0.018$. Thus the confidence interval for the mean is approximately $\bar{Y} \pm SE_{\bar{Y}} t_{0.05(2),200} = 1.5 \pm (0.018)(1.97) = 1.5 \pm 0.036$.