Appendix 3. Statistical tables

This appendix gives numerical values for a few of the most commonly used probability distributions. More can be found in references such as Rohlf and Sokal, *Biostatistical Tables*.

Table 1: χ^2 distributions

Table 2: *Z* distribution

Table 3: Student's *t* distribution

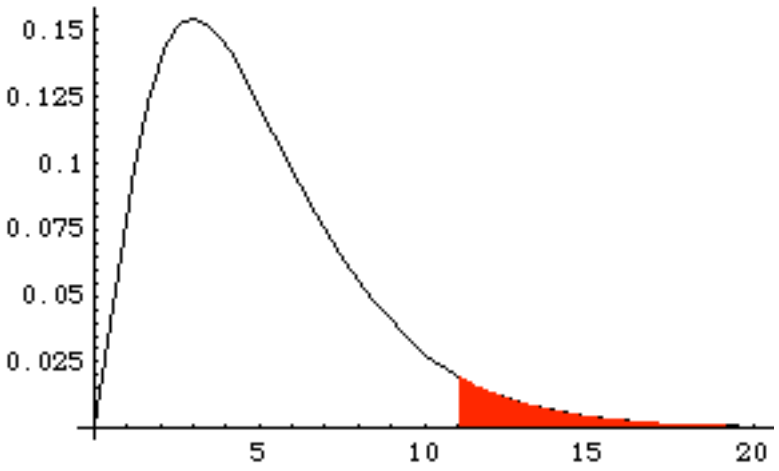
Table 4: *F* distribution

Table 5: Mann-Whitney U distribution

Table 6: The correlation coefficient, *r*.

Statistical Table 1. The χ^2 distribution

This table gives value of a χ^2 distribution above which there is X of the probability. X is given on the top row, and the number of degrees of freedom is given in the far left column.



For example, in this graph, 5% of the probability is in the red section. There are 5 degrees of freedom, and the boundary of the red section starts at $\chi^2 = 11.07$.

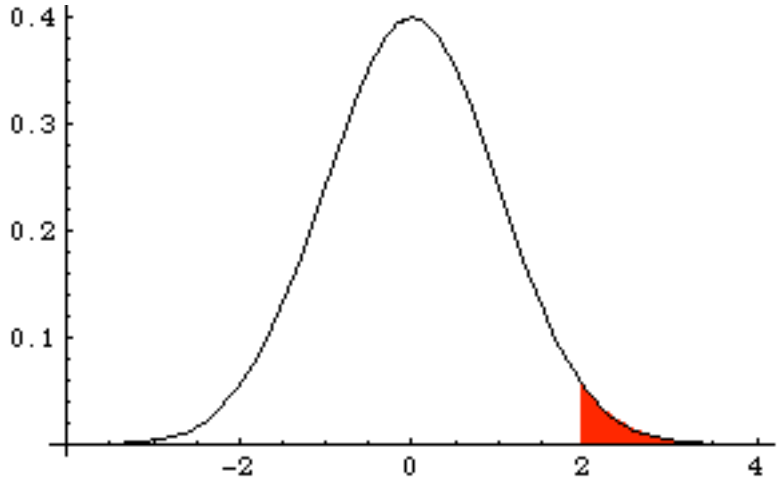
df \ *X*	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001
1	1.6 E-6	3.9E-5	0.00016	0.00098	0.00393	3.84	5.02	6.63	7.88	10.83
2	0.002	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.6	13.82
3	0.02	0.07	0.11	0.22	0.35	7.81	9.35	11.34	12.84	16.27
4	0.09	0.21	0.30	0.48	0.71	9.49	11.14	13.28	14.86	18.47
5	0.21	0.41	0.55	0.83	1.15	11.07	12.83	15.09	16.75	20.52
6	0.38	0.68	0.87	1.24	1.64	12.59	14.45	16.81	18.55	22.46
7	0.60	0.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28	24.32
8	0.86	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.95	26.12
9	1.15	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59	27.88
10	1.48	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19	29.59
11	1.83	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76	31.26
12	2.21	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.3	32.91
13	2.62	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82	34.53
14	3.04	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32	36.12
15	3.48	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80	37.70
16	3.94	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27	39.25
17	4.42	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72	40.79
18	4.90	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16	42.31
19	5.41	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58	43.82
20	5.92	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00	45.31
21	6.45	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.4	46.80
22	6.98	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80	48.27
23	7.53	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18	49.73
24	8.08	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56	51.18
25	8.65	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93	52.62
26	9.22	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29	54.05
27	9.80	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64	55.48
28	10.39	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99	56.89
29	10.99	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34	58.30
30	11.59	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67	59.70
31	12.20	14.46	15.66	17.54	19.28	44.99	48.23	52.19	55.00	61.10
32	12.81	15.13	16.36	18.29	20.07	46.19	49.48	53.49	56.33	62.49
33	13.43	15.82	17.07	19.05	20.87	47.40	50.73	54.78	57.65	63.87
34	14.06	16.50	17.79	19.81	21.66	48.60	51.97	56.06	58.96	65.25
35	14.69	17.19	18.51	20.57	22.47	49.80	53.20	57.34	60.27	66.62
36	15.32	17.89	19.23	21.34	23.27	51.00	54.44	58.62	61.58	67.99
37	15.97	18.59	19.96	22.11	24.07	52.19	55.67	59.89	62.88	69.35
38	16.61	19.29	20.69	22.88	24.88	53.38	56.90	61.16	64.18	70.70
39	17.26	20.00	21.43	23.65	25.70	54.57	58.12	62.43	65.48	72.05
40	17.92	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77	73.40

df \ *X*	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001
41	18.58	21.42	22.91	25.21	27.33	56.94	60.56	64.95	68.05	74.74
42	19.24	22.14	23.65	26.00	28.14	58.12	61.78	66.21	69.34	76.08
43	19.91	22.86	24.40	26.79	28.96	59.30	62.99	67.46	70.62	77.42
44	20.58	23.58	25.15	27.57	29.79	60.48	64.20	68.71	71.89	78.75
45	21.25	24.31	25.90	28.37	30.61	61.66	65.41	69.96	73.17	80.08
46	21.93	25.04	26.66	29.16	31.44	62.83	66.62	71.20	74.44	81.40
47	22.61	25.77	27.42	29.96	32.27	64.00	67.82	72.44	75.70	82.72
48	23.29	26.51	28.18	30.75	33.10	65.17	69.02	73.68	76.97	84.04
49	23.98	27.25	28.94	31.55	33.93	66.34	70.22	74.92	78.23	85.35
50	24.67	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49	86.66
51	25.37	28.73	30.48	33.16	35.60	68.67	72.62	77.39	80.75	87.97
52	26.07	29.48	31.25	33.97	36.44	69.83	73.81	78.62	82.00	89.27
53	26.76	30.23	32.02	34.78	37.28	70.99	75.00	79.84	83.25	90.57
54	27.47	30.98	32.79	35.59	38.12	72.15	76.19	81.07	84.50	91.87
55	28.17	31.73	33.57	36.40	38.96	73.31	77.38	82.29	85.75	93.17
56	28.88	32.49	34.35	37.21	39.80	74.47	78.57	83.51	86.99	94.46
57	29.59	33.25	35.13	38.03	40.65	75.62	79.75	84.73	88.24	95.75
58	30.30	34.01	35.91	38.84	41.49	76.78	80.94	85.95	89.48	97.04
59	31.02	34.77	36.70	39.66	42.34	77.93	82.12	87.17	90.72	98.32
60	31.74	35.53	37.48	40.48	43.19	79.08	83.30	88.38	91.95	99.61
61	32.46	36.30	38.27	41.30	44.04	80.23	84.48	89.59	93.19	100.89
62	33.18	37.07	39.06	42.13	44.89	81.38	85.65	90.80	94.42	102.17
63	33.91	37.84	39.86	42.95	45.74	82.53	86.83	92.01	95.65	103.44
64	34.63	38.61	40.65	43.78	46.59	83.68	88.00	93.22	96.88	104.72
65	35.36	39.38	41.44	44.60	47.45	84.82	89.18	94.42	98.11	105.99
66	36.09	40.16	42.24	45.43	48.31	85.96	90.35	95.63	99.33	107.26
67	36.83	40.94	43.04	46.26	49.16	87.11	91.52	96.83	100.55	108.53
68	37.56	41.71	43.84	47.09	50.02	88.25	92.69	98.03	101.78	109.79
69	38.30	42.49	44.64	47.92	50.88	89.39	93.86	99.23	103.00	111.06
70	39.04	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.21	112.32
71	39.78	44.06	46.25	49.59	52.60	91.67	96.19	101.62	105.43	113.58
72	40.52	44.84	47.05	50.43	53.46	92.81	97.35	102.82	106.65	114.84
73	41.26	45.63	47.86	51.26	54.33	93.95	98.52	104.01	107.86	116.09
74	42.01	46.42	48.67	52.10	55.19	95.08	99.68	105.20	109.07	117.35
75	42.76	47.21	49.48	52.94	56.05	96.22	100.84	106.39	110.29	118.60
76	43.51	48.00	50.29	53.78	56.92	97.35	102.00	107.58	111.50	119.85
77	44.26	48.79	51.10	54.62	57.79	98.48	103.16	108.77	112.70	121.10
78	45.01	49.58	51.91	55.47	58.65	99.62	104.32	109.96	113.91	122.35
79	45.76	50.38	52.72	56.31	59.52	100.75	105.47	111.14	115.12	123.59
80	46.52	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.32	124.84
81	47.28	51.97	54.36	58.00	61.26	103.01	107.78	113.51	117.52	126.08
82	48.04	52.77	55.17	58.84	62.13	104.14	108.94	114.69	118.73	127.32
83	48.80	53.57	55.99	59.69	63.00	105.27	110.09	115.88	119.93	128.56

df \ *X*	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001
84	49.56	54.37	56.81	60.54	63.88	106.39	111.24	117.06	121.13	129.80
85	50.32	55.17	57.63	61.39	64.75	107.52	112.39	118.24	122.32	131.04
86	51.08	55.97	58.46	62.24	65.62	108.65	113.54	119.41	123.52	132.28
87	51.85	56.78	59.28	63.09	66.50	109.77	114.69	120.59	124.72	133.51
88	52.62	57.58	60.10	63.94	67.37	110.90	115.84	121.77	125.91	134.75
89	53.39	58.39	60.93	64.79	68.25	112.02	116.99	122.94	127.11	135.98
90	54.16	59.20	61.75	65.65	69.13	113.15	118.14	124.12	128.30	137.21
91	54.93	60.00	62.58	66.50	70.00	114.27	119.28	125.29	129.49	138.44
92	55.70	60.81	63.41	67.36	70.88	115.39	120.43	126.46	130.68	139.67
93	56.47	61.63	64.24	68.21	71.76	116.51	121.57	127.63	131.87	140.89
94	57.25	62.44	65.07	69.07	72.64	117.63	122.72	128.80	133.06	142.12
95	58.02	63.25	65.90	69.92	73.52	118.75	123.86	129.97	134.25	143.34
96	58.80	64.06	66.73	70.78	74.40	119.87	125.00	131.14	135.43	144.57
97	59.58	64.88	67.56	71.64	75.28	120.99	126.14	132.31	136.62	145.79
98	60.36	65.69	68.40	72.50	76.16	122.11	127.28	133.48	137.80	147.01
99	61.14	66.51	69.23	73.36	77.05	123.23	128.42	134.64	138.99	148.23
100	61.92	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.17	149.45

Statistical Table 2. The standard normal (*Z*) distribution

The far left column gives the first two digits of Z, while the top row gives the last digit. The number in the table itself gives the probability of a standard normal deviate being greater than this number. For example, to find the probability of a Z greater than 1.96, we would go down to the row starting with 1.9 and on that row scan over to the column under x.x6. We would find the number 0.025, meaning that 2.5% of the time a standard normal deviate is greater than 1.96.



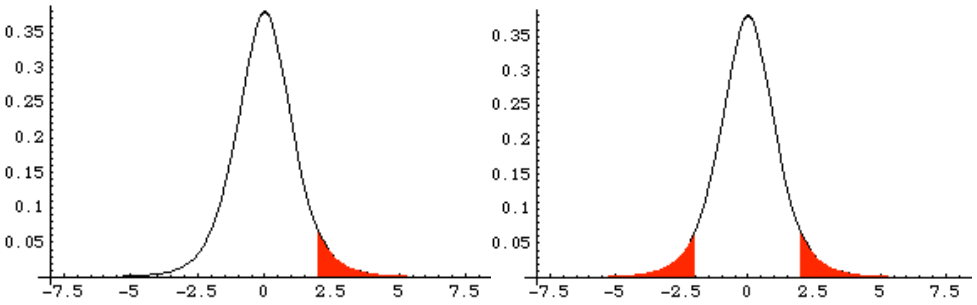
	x.x0	x.x1	x.x2	.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
0.0	0.5	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.1335	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639

	x.x0	x.x1	x.x2	.x3	x.x4	x.x5	x.x6	x.x7	x.x8	x.x9
2.5	0.00621	0.00604	0.00587	0.0057	0.00554	0.00539	0.00523	0.00508	0.00494	0.0048
2.6	0.00466	0.00453	0.0044	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.0028	0.00272	0.00264
2.8	0.00256	0.00248	0.0024	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.001
3.1	0.00097	0.00094	0.0009	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.0006	0.00058	0.00056	0.00054	0.00052	0.0005
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.0004	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.0003	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.0002	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.0001	0.0001	0.0001	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002

Statistical Table 3. The Student *t* distribution

This table gives value of a *t* distribution above which there is $\alpha(1)$ of the probability. α is given on the top row, and the number of degrees of freedom is given in the far left column. For two-tailed tests, the critical value given for $\alpha(1)$ is the same as $\alpha(2) = 2\ \alpha(1)$.

For example, with 5 degrees of freedom, 5% of the probability is above t=2.02, and 10% of the probability is either above 2.02 or below -2.02.



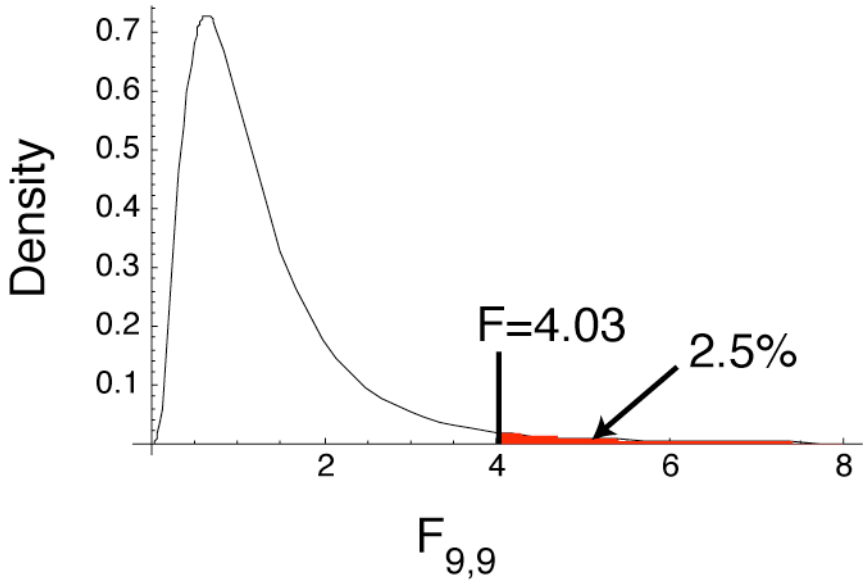
df	α(1) =0.1 α(2)=0.2	α(1) =0.05 α(2)=0.10	α(1) =0.025 α(2)=0.05	α(1) =0.01 α(2)=0.02	α(1) =0.005 α(2)=0.01	α(1) =0.001 α(2)=0.002	α(1) =0.0001 α(2)=0.0002
1	3.08	6.31	12.71	31.82	63.66	318.31	3183.1
2	1.89	2.92	4.30	6.96	9.92	22.33	70.70
3	1.64	2.35	3.18	4.54	5.84	10.21	22.20
4	1.53	2.13	2.78	3.75	4.60	7.17	13.03
5	1.48	2.02	2.57	3.36	4.03	5.89	9.68
6	1.44	1.94	2.45	3.14	3.71	5.21	8.02
7	1.41	1.89	2.36	3.00	3.50	4.79	7.06
8	1.40	1.86	2.31	2.90	3.36	4.50	6.44
9	1.38	1.83	2.26	2.82	3.25	4.30	6.01
10	1.37	1.81	2.23	2.76	3.17	4.14	5.69
11	1.36	1.80	2.20	2.72	3.11	4.02	5.45
12	1.36	1.78	2.18	2.68	3.05	3.93	5.26
13	1.35	1.77	2.16	2.65	3.01	3.85	5.11
14	1.35	1.76	2.14	2.62	2.98	3.79	4.99
15	1.34	1.75	2.13	2.60	2.95	3.73	4.88
16	1.34	1.75	2.12	2.58	2.92	3.69	4.79
17	1.33	1.74	2.11	2.57	2.90	3.65	4.71
18	1.33	1.73	2.10	2.55	2.88	3.61	4.65
19	1.33	1.73	2.09	2.54	2.86	3.58	4.59
20	1.33	1.72	2.09	2.53	2.85	3.55	4.54
21	1.32	1.72	2.08	2.52	2.83	3.53	4.49
22	1.32	1.72	2.07	2.51	2.82	3.50	4.45
23	1.32	1.71	2.07	2.50	2.81	3.48	4.42
24	1.32	1.71	2.06	2.49	2.80	3.47	4.38
25	1.32	1.71	2.06	2.49	2.79	3.45	4.35
26	1.31	1.71	2.06	2.48	2.78	3.43	4.32
27	1.31	1.70	2.05	2.47	2.77	3.42	4.30
28	1.31	1.70	2.05	2.47	2.76	3.41	4.28
29	1.31	1.70	2.05	2.46	2.76	3.40	4.25
30	1.31	1.70	2.04	2.46	2.75	3.39	4.23
31	1.31	1.70	2.04	2.45	2.74	3.37	4.22
32	1.31	1.69	2.04	2.45	2.74	3.37	4.20
33	1.31	1.69	2.03	2.44	2.73	3.36	4.18
34	1.31	1.69	2.03	2.44	2.73	3.35	4.17
35	1.31	1.69	2.03	2.44	2.72	3.34	4.15
36	1.31	1.69	2.03	2.43	2.72	3.33	4.14
37	1.30	1.69	2.03	2.43	2.72	3.33	4.13
38	1.30	1.69	2.02	2.43	2.71	3.32	4.12
39	1.30	1.68	2.02	2.43	2.71	3.31	4.10

df	α(1) =0.1 α(2)=0.2	α(1) =0.05 α(2)=0.10	α(1) =0.025 α(2)=0.05	α(1) =0.01 α(2)=0.02	α(1) =0.005 α(2)=0.01	α(1) =0.001 α(2)=0.002	α(1) =0.0001 α(2)=0.0002
40	1.30	1.68	2.02	2.42	2.70	3.31	4.09
41	1.30	1.68	2.02	2.42	2.70	3.30	4.08
42	1.30	1.68	2.02	2.42	2.70	3.30	4.07
43	1.30	1.68	2.02	2.42	2.70	3.29	4.07
44	1.30	1.68	2.02	2.41	2.69	3.29	4.06
45	1.30	1.68	2.01	2.41	2.69	3.28	4.05
46	1.30	1.68	2.01	2.41	2.69	3.28	4.04
47	1.30	1.68	2.01	2.41	2.68	3.27	4.03
48	1.30	1.68	2.01	2.41	2.68	3.27	4.03
49	1.30	1.68	2.01	2.40	2.68	3.27	4.02
50	1.30	1.68	2.01	2.40	2.68	3.26	4.01
51	1.30	1.68	2.01	2.40	2.68	3.26	4.01
52	1.30	1.67	2.01	2.40	2.67	3.25	4.00
53	1.30	1.67	2.01	2.40	2.67	3.25	4.00
54	1.30	1.67	2.00	2.40	2.67	3.25	3.99
55	1.30	1.67	2.00	2.40	2.67	3.25	3.99
56	1.30	1.67	2.00	2.39	2.67	3.24	3.98
57	1.30	1.67	2.00	2.39	2.66	3.24	3.98
58	1.30	1.67	2.00	2.39	2.66	3.24	3.97
59	1.30	1.67	2.00	2.39	2.66	3.23	3.97
60	1.30	1.67	2.00	2.39	2.66	3.23	3.96
61	1.30	1.67	2.00	2.39	2.66	3.23	3.96
62	1.30	1.67	2.00	2.39	2.66	3.23	3.95
63	1.30	1.67	2.00	2.39	2.66	3.22	3.95
64	1.29	1.67	2.00	2.39	2.65	3.22	3.95
65	1.29	1.67	2.00	2.39	2.65	3.22	3.94
66	1.29	1.67	2.00	2.38	2.65	3.22	3.94
67	1.29	1.67	2.00	2.38	2.65	3.22	3.94
68	1.29	1.67	2.00	2.38	2.65	3.21	3.93
69	1.29	1.67	1.99	2.38	2.65	3.21	3.93
70	1.29	1.67	1.99	2.38	2.65	3.21	3.93
71	1.29	1.67	1.99	2.38	2.65	3.21	3.92
72	1.29	1.67	1.99	2.38	2.65	3.21	3.92
73	1.29	1.67	1.99	2.38	2.64	3.21	3.92
74	1.29	1.67	1.99	2.38	2.64	3.20	3.91
75	1.29	1.67	1.99	2.38	2.64	3.20	3.91
76	1.29	1.67	1.99	2.38	2.64	3.20	3.91
77	1.29	1.66	1.99	2.38	2.64	3.20	3.91
78	1.29	1.66	1.99	2.38	2.64	3.20	3.90
79	1.29	1.66	1.99	2.37	2.64	3.20	3.90
80	1.29	1.66	1.99	2.37	2.64	3.20	3.90
81	1.29	1.66	1.99	2.37	2.64	3.19	3.90

df	α(1) =0.1 α(2)=0.2	α(1) =0.05 α(2)=0.10	α(1) =0.025 α(2)=0.05	α(1) =0.01 α(2)=0.02	α(1) =0.005 α(2)=0.01	α(1) =0.001 α(2)=0.002	α(1) =0.0001 α(2)=0.0002
82	1.29	1.66	1.99	2.37	2.64	3.19	3.89
83	1.29	1.66	1.99	2.37	2.64	3.19	3.89
84	1.29	1.66	1.99	2.37	2.64	3.19	3.89
85	1.29	1.66	1.99	2.37	2.63	3.19	3.89
86	1.29	1.66	1.99	2.37	2.63	3.19	3.89
87	1.29	1.66	1.99	2.37	2.63	3.19	3.88
88	1.29	1.66	1.99	2.37	2.63	3.19	3.88
89	1.29	1.66	1.99	2.37	2.63	3.18	3.88
90	1.29	1.66	1.99	2.37	2.63	3.18	3.88
100	1.29	1.66	1.98	2.36	2.63	3.17	3.86
120	1.29	1.66	1.98	2.36	2.62	3.16	3.84
140	1.29	1.66	1.98	2.35	2.61	3.15	3.82
160	1.29	1.65	1.97	2.35	2.61	3.14	3.81
180	1.29	1.65	1.97	2.35	2.60	3.14	3.80
200	1.29	1.65	1.97	2.35	2.60	3.13	3.79
400	1.28	1.65	1.97	2.34	2.59	3.11	3.75
1000	1.28	1.65	1.96	2.33	2.58	3.10	3.73

Statistical Table 4. The *F* distribution

These tables give critical values for α =0.05 and α= 0.025 for a range of numerator degrees of freedom. For these tables, all numbers on the same page correspond to the same α value, with numerator *df* listed across the top row and denominator *df* given in the first column of each other row.



Critical value of F, $\alpha(1)$=0.05, $\alpha(2)$=0.10

	Numerator *df*									
den. *df*	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85

Critical value of *F,* $\alpha(1)$=0.05, $\alpha(2)$=0.10, continued

	Numerator *df*									
den. *df*	12	15	20	30	40	60	100	200	400	1000
1	243.91	245.95	248.01	250.10	251.14	252.20	253.04	253.68	254.00	254.19
2	19.41	19.43	19.45	19.46	19.47	19.48	19.49	19.49	19.49	19.49
3	8.74	8.70	8.66	8.62	8.59	8.57	8.55	8.54	8.53	8.53
4	5.91	5.86	5.80	5.75	5.72	5.69	5.66	5.65	5.64	5.63
5	4.68	4.62	4.56	4.50	4.46	4.43	4.41	4.39	4.38	4.37
6	4.00	3.94	3.87	3.81	3.77	3.74	3.71	3.69	3.68	3.67
7	3.57	3.51	3.44	3.38	3.34	3.30	3.27	3.25	3.24	3.23
8	3.28	3.22	3.15	3.08	3.04	3.01	2.97	2.95	2.94	2.93
9	3.07	3.01	2.94	2.86	2.83	2.79	2.76	2.73	2.72	2.71
10	2.91	2.85	2.77	2.70	2.66	2.62	2.59	2.56	2.55	2.54
11	2.79	2.72	2.65	2.57	2.53	2.49	2.46	2.43	2.42	2.41
12	2.69	2.62	2.54	2.47	2.43	2.38	2.35	2.32	2.31	2.30
13	2.60	2.53	2.46	2.38	2.34	2.30	2.26	2.23	2.22	2.21
14	2.53	2.46	2.39	2.31	2.27	2.22	2.19	2.16	2.15	2.14
15	2.48	2.40	2.33	2.25	2.20	2.16	2.12	2.10	2.08	2.07
16	2.42	2.35	2.28	2.19	2.15	2.11	2.07	2.04	2.02	2.02
17	2.38	2.31	2.23	2.15	2.10	2.06	2.02	1.99	1.98	1.97
18	2.34	2.27	2.19	2.11	2.06	2.02	1.98	1.95	1.93	1.92
19	2.31	2.23	2.16	2.07	2.03	1.98	1.94	1.91	1.89	1.88
20	2.28	2.20	2.12	2.04	1.99	1.95	1.91	1.88	1.86	1.85
21	2.25	2.18	2.10	2.01	1.96	1.92	1.88	1.84	1.83	1.82
22	2.23	2.15	2.07	1.98	1.94	1.89	1.85	1.82	1.80	1.79
23	2.20	2.13	2.05	1.96	1.91	1.86	1.82	1.79	1.77	1.76
24	2.18	2.11	2.03	1.94	1.89	1.84	1.80	1.77	1.75	1.74
25	2.16	2.09	2.01	1.92	1.87	1.82	1.78	1.75	1.73	1.72
26	2.15	2.07	1.99	1.90	1.85	1.80	1.76	1.73	1.71	1.70
27	2.13	2.06	1.97	1.88	1.84	1.79	1.74	1.71	1.69	1.68
28	2.12	2.04	1.96	1.87	1.82	1.77	1.73	1.69	1.67	1.66
29	2.10	2.03	1.94	1.85	1.81	1.75	1.71	1.67	1.66	1.65
30	2.09	2.01	1.93	1.84	1.79	1.74	1.70	1.66	1.64	1.63
40	2.00	1.92	1.84	1.74	1.69	1.64	1.59	1.55	1.53	1.52
50	1.95	1.87	1.78	1.69	1.63	1.58	1.52	1.48	1.46	1.45
60	1.92	1.84	1.75	1.65	1.59	1.53	1.48	1.44	1.41	1.40
70	1.89	1.81	1.72	1.62	1.57	1.50	1.45	1.40	1.38	1.36
80	1.88	1.79	1.70	1.60	1.54	1.48	1.43	1.38	1.35	1.34
90	1.86	1.78	1.69	1.59	1.53	1.46	1.41	1.36	1.33	1.31
100	1.85	1.77	1.68	1.57	1.52	1.45	1.39	1.34	1.31	1.30
200	1.80	1.72	1.62	1.52	1.46	1.39	1.32	1.26	1.23	1.21
400	1.78	1.69	1.60	1.49	1.42	1.35	1.28	1.22	1.18	1.15

Critical value of *F*, $\alpha(1)$=0.025, $\alpha(2)$=0.05

	Numerator *df*									
den. *df*	1	2	3	4	5	6	7	8	9	10
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
2	38.51	39.00	39.17	39.25	39.30	3 9 . 3 3	39.36	39.37	39.39	39.40
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39
50	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38	2.32
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27
70	5.25	3.89	3.31	2.97	2.75	2.59	2.47	2.38	2.30	2.24
80	5.22	3.86	3.28	2.95	2.73	2.57	2.45	2.35	2.28	2.21
90	5.20	3.84	3.26	2.93	2.71	2.55	2.43	2.34	2.26	2.19
100	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18
200	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18	2.11
400	5.06	3.72	3.15	2.82	2.60	2.44	2.32	2.22	2.15	2.08

Critical value of F, $\alpha(1)$=0.025, $\alpha(2)$=0.05, continued

	Numerator *df*									
den. *df*	12	15	20	30	40	60	100	200	400	1000
1	976.71	984.87	993.10	1001.40	1005.60	1009.80	1013.20	1015.70	1017.00	1017.80
2	39.41	39.43	39.45	39.46	39.47	39.48	39.49	39.49	39.50	39.50
3	14.34	14.25	14.17	14.08	14.04	13.99	13.96	13.93	13.92	13.91
4	8.75	8.66	8.56	8.46	8.41	8.36	8.32	8.29	8.27	8.26
5	6.52	6.43	6.33	6.23	6.18	6.12	6.08	6.05	6.03	6.02
6	5.37	5.27	5.17	5.07	5.01	4.96	4.92	4.88	4.87	4.86
7	4.67	4.57	4.47	4.36	4.31	4.25	4.21	4.18	4.16	4.15
8	4.20	4.10	4.00	3.89	3.84	3.78	3.74	3.70	3.69	3.68
9	3.87	3.77	3.67	3.56	3.51	3.45	3.40	3.37	3.35	3.34
10	3.62	3.52	3.42	3.31	3.26	3.20	3.15	3.12	3.10	3.09
11	3.43	3.33	3.23	3.12	3.06	3.00	2.96	2.92	2.90	2.89
12	3.28	3.18	3.07	2.96	2.91	2.85	2.80	2.76	2.74	2.73
13	3.15	3.05	2.95	2.84	2.78	2.72	2.67	2.63	2.61	2.60
14	3.05	2.95	2.84	2.73	2.67	2.61	2.56	2.53	2.51	2.50
15	2.96	2.86	2.76	2.64	2.59	2.52	2.47	2.44	2.42	2.40
16	2.89	2.79	2.68	2.57	2.51	2.45	2.40	2.36	2.34	2.32
17	2.82	2.72	2.62	2.50	2.44	2.38	2.33	2.29	2.27	2.26
18	2.77	2.67	2.56	2.44	2.38	2.32	2.27	2.23	2.21	2.20
19	2.72	2.62	2.51	2.39	2.33	2.27	2.22	2.18	2.15	2.14
20	2.68	2.57	2.46	2.35	2.29	2.22	2.17	2.13	2.11	2.09
21	2.64	2.53	2.42	2.31	2.25	2.18	2.13	2.09	2.06	2.05
22	2.60	2.50	2.39	2.27	2.21	2.14	2.09	2.05	2.03	2.01
23	2.57	2.47	2.36	2.24	2.18	2.11	2.06	2.01	1.99	1.98
24	2.54	2.44	2.33	2.21	2.15	2.08	2.02	1.98	1.96	1.94
25	2.51	2.41	2.30	2.18	2.12	2.05	2.00	1.95	1.93	1.91
26	2.49	2.39	2.28	2.16	2.09	2.03	1.97	1.92	1.90	1.89
27	2.47	2.36	2.25	2.13	2.07	2.00	1.94	1.90	1.88	1.86
28	2.45	2.34	2.23	2.11	2.05	1.98	1.92	1.88	1.85	1.84
29	2.43	2.32	2.21	2.09	2.03	1.96	1.90	1.86	1.83	1.82
30	2.41	2.31	2.20	2.07	2.01	1.94	1.88	1.84	1.81	1.80
40	2.29	2.18	2.07	1.94	1.88	1.80	1.74	1.69	1.66	1.65
50	2.22	2.11	1.99	1.87	1.80	1.72	1.66	1.60	1.57	1.56
60	2.17	2.06	1.94	1.82	1.74	1.67	1.60	1.54	1.51	1.49
70	2.14	2.03	1.91	1.78	1.71	1.63	1.56	1.50	1.47	1.45
80	2.11	2.00	1.88	1.75	1.68	1.60	1.53	1.47	1.43	1.41
90	2.09	1.98	1.86	1.73	1.66	1.58	1.50	1.44	1.41	1.39
100	2.08	1.97	1.85	1.71	1.64	1.56	1.48	1.42	1.39	1.36
200	2.01	1.90	1.78	1.64	1.56	1.47	1.39	1.32	1.28	1.25
400	1.98	1.87	1.74	1.60	1.52	1.43	1.35	1.27	1.22	1.18

Statistical table 5: Mann-Whitney U distribution

When the sample size increases above 10 for either sample, the *Z* approximation given in the text works reasonably well. Here we give reduced version of the tables for the Mann-Whitney *U* distribution. Test statistics larger than those given in the table will be significant at the given *a* level. n_1 and $n_2$refer to the sample sizes of the two samples. "-" means that it is not possible to reject a null hypothesis with that α with those sample sizes.

U, $\alpha = 0.05$

n_1

n_2	3	4	5	6	7	8	9	10
3	-	-	15	17	20	22	25	27
4	-	16	19	22	25	28	32	35
5	15	19	23	27	30	34	38	42
6	17	22	27	31	36	40	44	49
7	20	25	30	36	41	46	51	56
8	22	28	34	40	46	51	57	63
9	25	32	38	44	51	57	64	70
10	27	35	42	49	56	63	70	77

U, α= 0.01,

n_1

n_2	3	4	5	6	7	8	9	10
3	-	-	-	-	-	-	27	30
4	-	-	-	24	28	31	35	38
5	-	-	25	29	34	38	42	46
6	-	24	29	34	39	44	49	54
7	-	28	34	39	45	50	56	61
8	-	31	38	44	50	57	63	69
9	27	35	42	49	56	63	70	77
10	30	38	46	54	61	69	77	84

Statistical table 6: *r*, the correlation coefficient

This table gives the values of *r* that correspond to the edge of the region that would be rejected by a two-tailed test. If *r* is greater than the given value, then $P < \alpha$ for a test of the null hypothesis that $\rho = 0$. The left-hand column gives the degrees of freedom of the test, which is *n* -1.

df	$\alpha(2) = 0.05$	$\alpha(2) = 0.01$
1	0.997	1.
2	0.950	0.990
3	0.878	0.959
4	0.811	0.917
5	0.754	0.875
6	0.707	0.834
7	0.666	0.798
8	0.632	0.765
9	0.602	0.735
10	0.576	0.708
11	0.553	0.684
12	0.532	0.661
13	0.514	0.641
14	0.497	0.623
15	0.482	0.606
16	0.468	0.59
17	0.456	0.575
18	0.444	0.561
19	0.433	0.549
20	0.423	0.537
21	0.413	0.526
22	0.404	0.515
23	0.396	0.505
24	0.388	0.496
25	0.381	0.487
26	0.374	0.479
27	0.367	0.471
28	0.361	0.463
29	0.355	0.456
30	0.349	0.449

df	$\alpha(2) = 0.05$	$\alpha(2) = 0.01$
35	0.325	0.418
40	0.304	0.393
45	0.288	0.372
50	0.273	0.354
55	0.261	0.339
60	0.250	0.325
70	0.232	0.302
80	0.217	0.283
90	0.205	0.267
100	0.195	0.254
200	0.138	0.181
300	0.113	0.148
400	0.098	0.128
500	0.088	0.115
600	0.080	0.105
700	0.074	0.097
800	0.069	0.091
900	0.065	0.086
1000	0.062	0.081
2000	0.044	0.058
5000	0.028	0.036