Departmental Ph.D. Oral
On the nature of the stock market: Simulations and experiments

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July 6, 2000

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1 Why study the stock market?

Market is a strongly-interacting, many-particle, non-equilibrium system. 'Nuff said.

Markets have surprising dynamics. Example: Were long believed to have normal fluctuations. Mandelbrot [1] discovered price fluctuations exhibited scaling: returns

$$r_\Delta(t) \equiv \log \frac{p(t)}{p(t-\Delta)} \tag{1}$$

have power law distribution tail

$$\Pr(r) \sim \frac{1}{r^{\alpha+1}} \tag{2}$$

over orders of magnitude of sampling interval Δ.

Scaling exponent is universal, $\alpha \approx 1.4$ [2].

Power law and universality are suggestive of a critical phase transition.

2 How does one study the stock market?

Analyze statistical properties of fluctuations. (Already been done.)

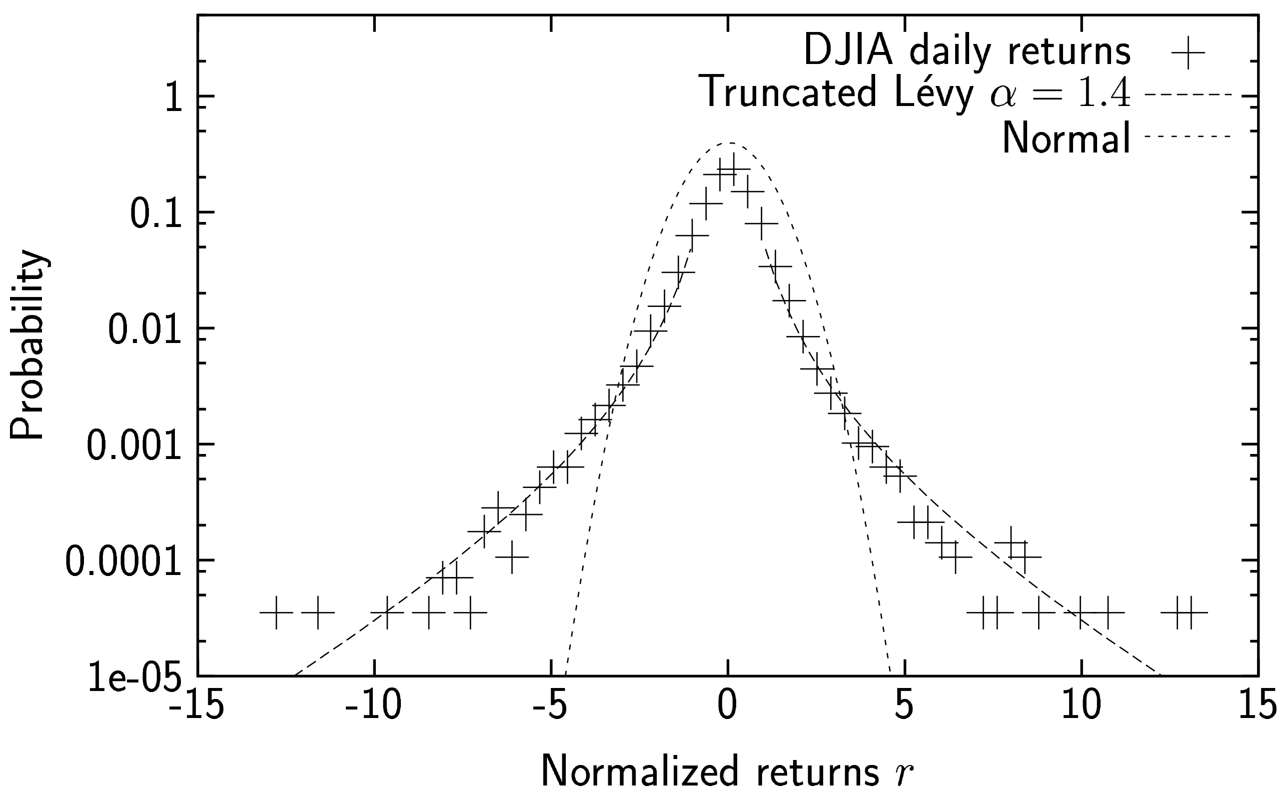
Try to build "microscopic" models which capture important features. (My approach.)

3 What is empirically known?

3.1 Fat tails

Distribution of returns falls off much more slowly than a Gaussian so frequency of large fluctuations much more common than might be expected.

For example, daily returns of the Dow Jones Industrial Average for the last hundred years [3]:

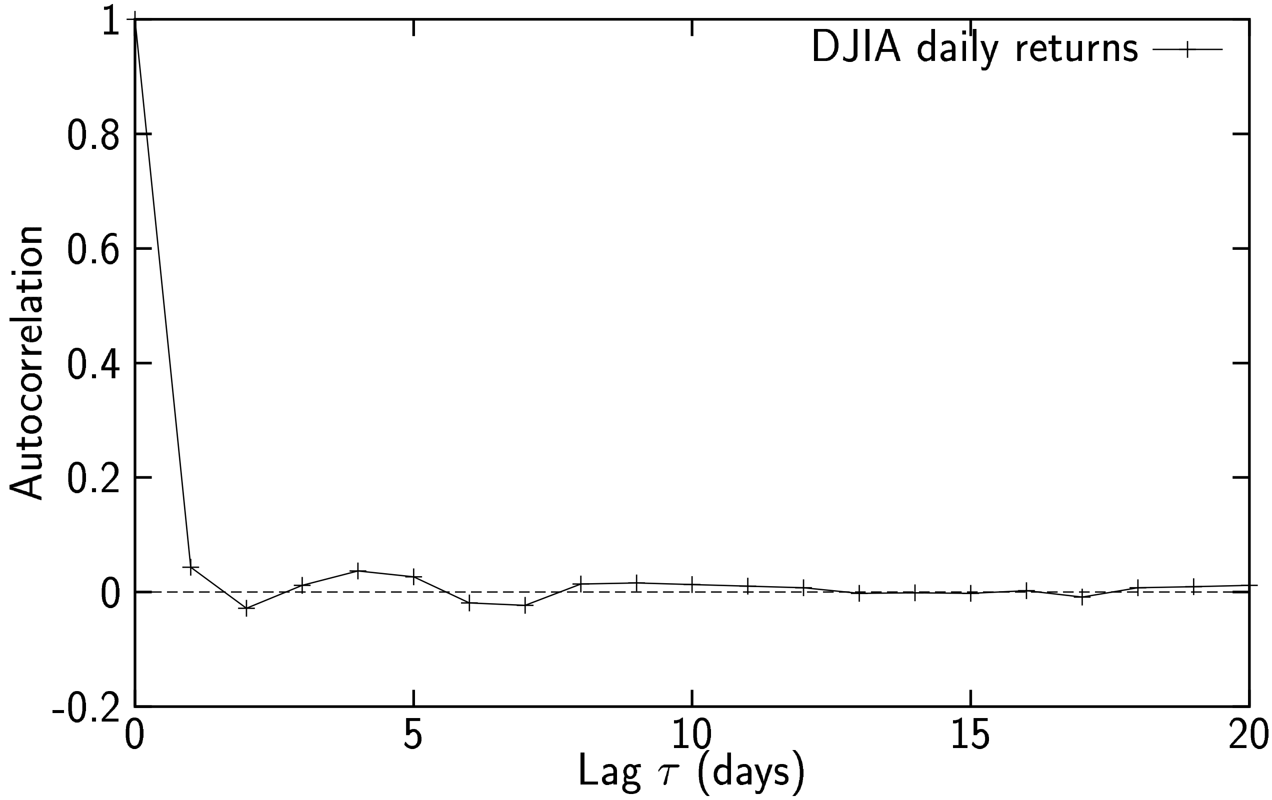


Best described by a truncated Lévy flight with power law tails which are attenuated.

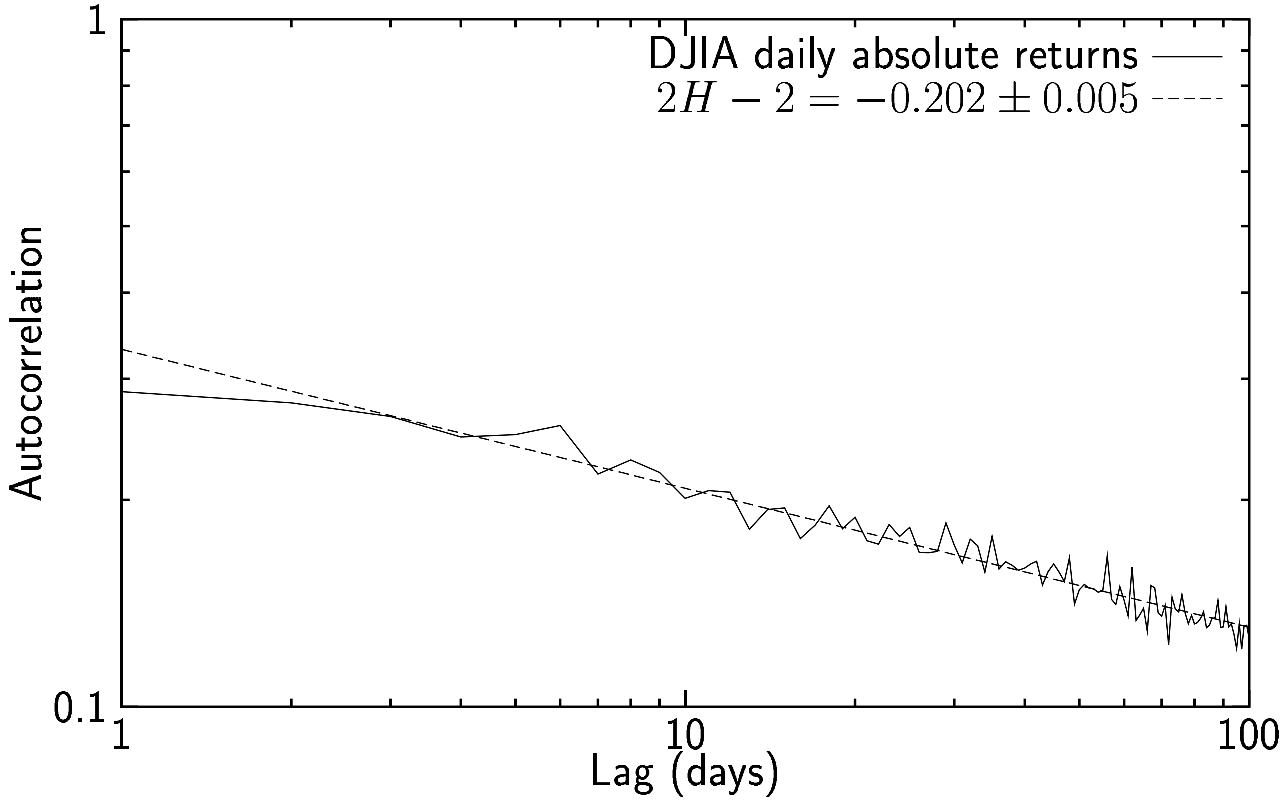
Frequency of a 10-standard deviation magnitude event is about once every 100 years, but for Gaussian is once every 10^{20} years.

3.2 Memory

No autocorrelation of returns. Can't predict movements from history:



But absolute values of returns are correlated; autocorrelation falls off as power law (much slower than exponential):



Hurst exponent $H \approx 0.9$ (very strong correlations).

Means large fluctuations tend to be clumped together—*clustered volatility*. Given a large movement can expect another one, but can't predict direction.

So the market does have a memory but can't use it to get rich!

4 How is the market modeled?

Built two models, CSEM and DSEM.

Both model N individual "agents" trading on a market with one stock.

Agents trade shares for cash and price emerges from their decisions.

Goal is to keep a certain fraction i of wealth invested in stock [4].

Models differ in how they choose i...

4.1 CSEM

Agents attempt to forecast tomorrow's return.

Forecast contains stochastic term ϵ. Chosen from Gaussian distribution with mean zero and standard deviation σ_ϵ (same for all agents).

$$\begin{aligned} \langle r_{t+1} \rangle_\epsilon &= \langle r_{t+1} \rangle + \epsilon_t &(3) \\ \mathrm{Var}\left[r_{t+1} \right]_\epsilon &= \mathrm{Var}\left[r_{t+1} \right] + \sigma_\epsilon^2. &(4) \end{aligned}$$

Submit trade orders to achieve investment fraction

$$i = \frac{\langle r_{t+1} \rangle_\epsilon}{a \mathrm{Var}\left[r_{t+1} \right]_\epsilon}. \tag{5}$$

(Subject to $0 < i < 1$.)

4.2 DSEM

Ideal investment fraction affected by stochastic news releases and price fluctuations. Define evidence [5]

$$e \equiv \log\left[\frac{i}{1-i}\right]. \tag{6}$$

Then on news release $\eta \sim N(0,1)$ evidence changes by

$$\Delta e \propto \eta \tag{7}$$

and on price change (log-return r)

$$\Delta e = r_p\, r. \tag{8}$$

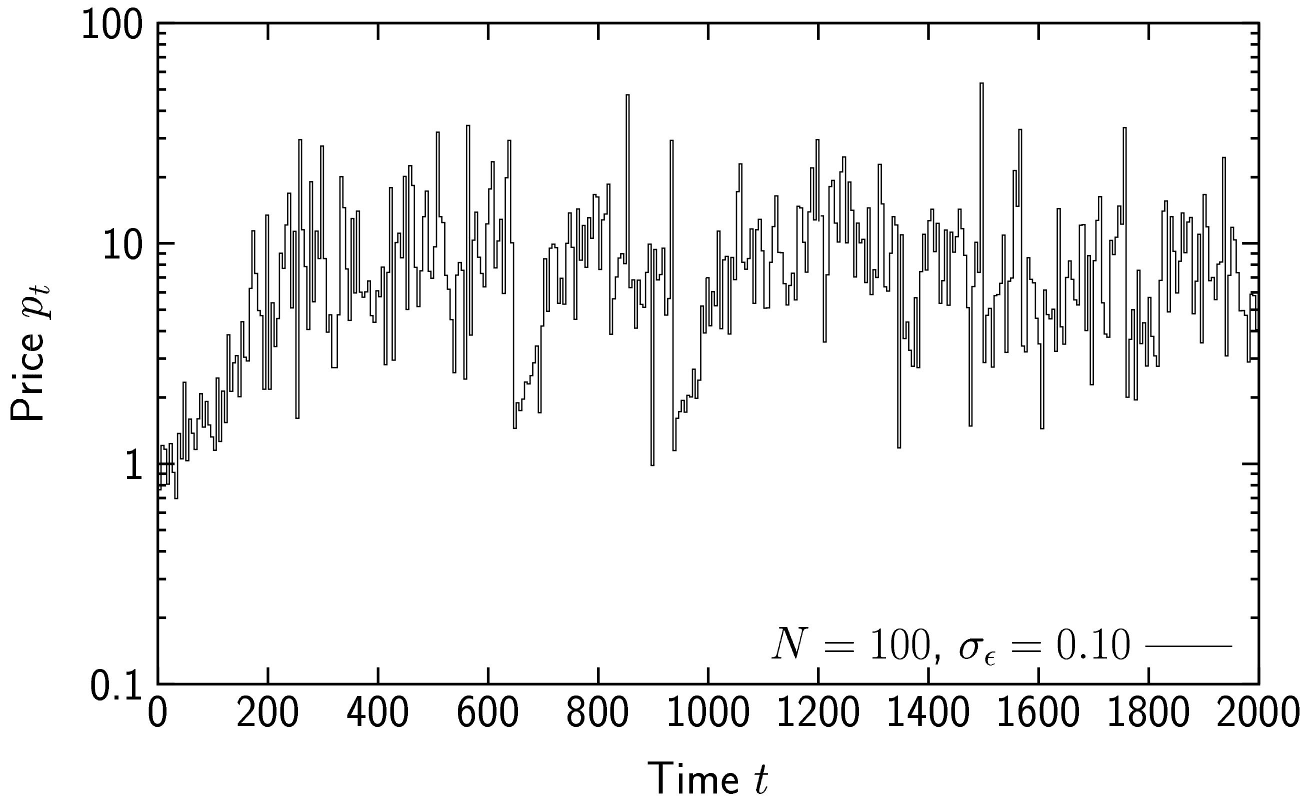
Strength of response to price movements characterized by parameter r_p. Like confidence in price movement. (Can be positive or negative.)

CSEM	DSEM
Centralized	Decentralized
(trade with market maker)	(trade with each other)
Price set by market maker	Price set by agents
Parallel updating	Poisson updating
Stochastic forecasts	Stochastic news events

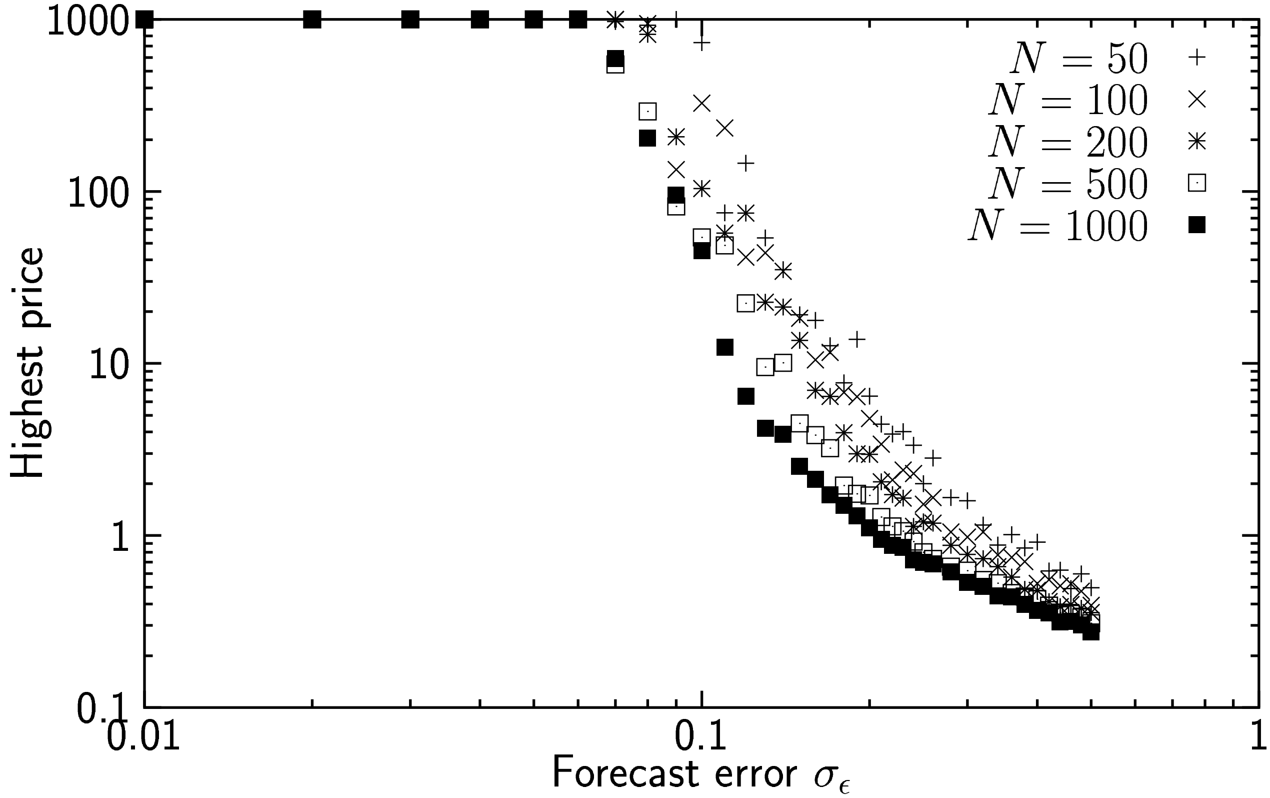
5 Where's the physics?

5.1 CSEM

After transient period price series tends to fluctuate around some steady-state with some maximum p_{max}:



As forecast error σ_ϵ drops, confidence in stock grows and price climbs:

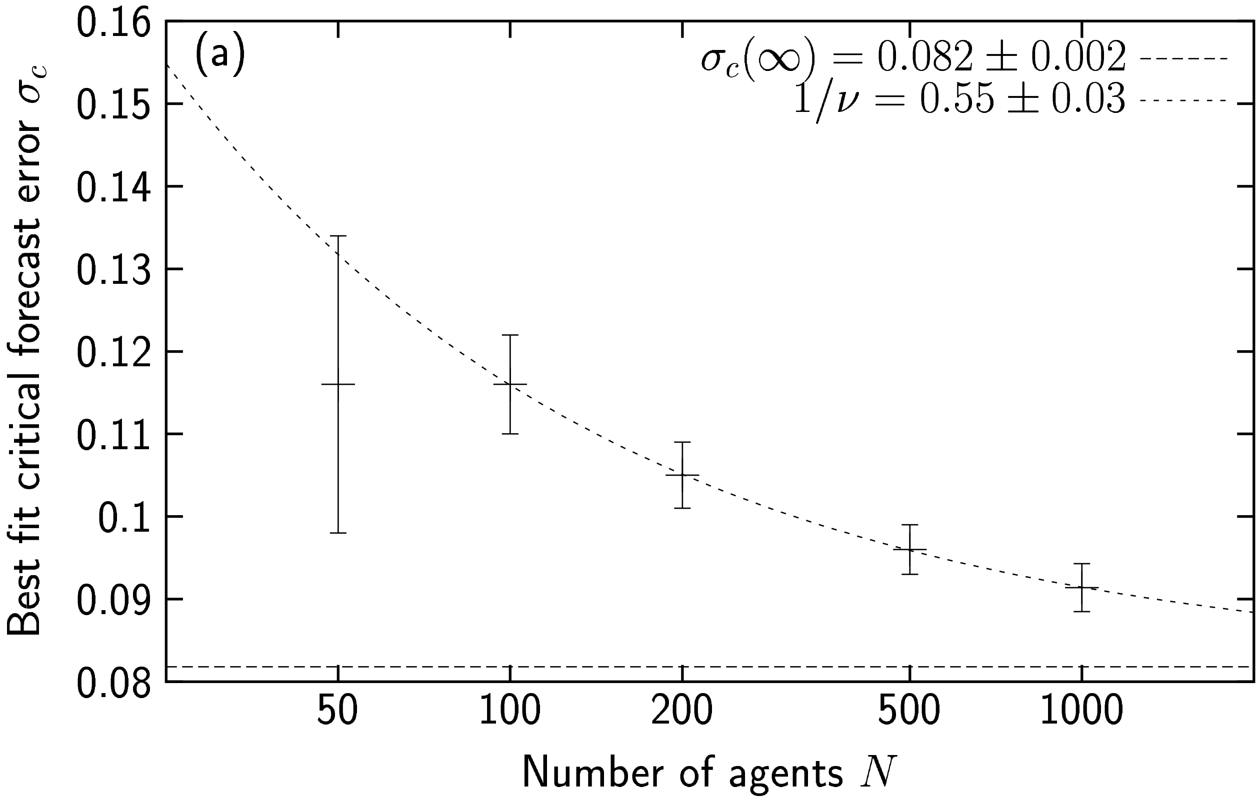


Price diverges at some non-zero $\sigma_\epsilon = \sigma_c$. Looks like critical point

$$p_{max} \propto (\sigma_\epsilon - \sigma_c)^{-b}. \tag{9}$$

Fitting to power law for various system sizes N gives finite-size scaling. Best fit exponent $b = 1.73 \pm 0.03$.

Critical point...

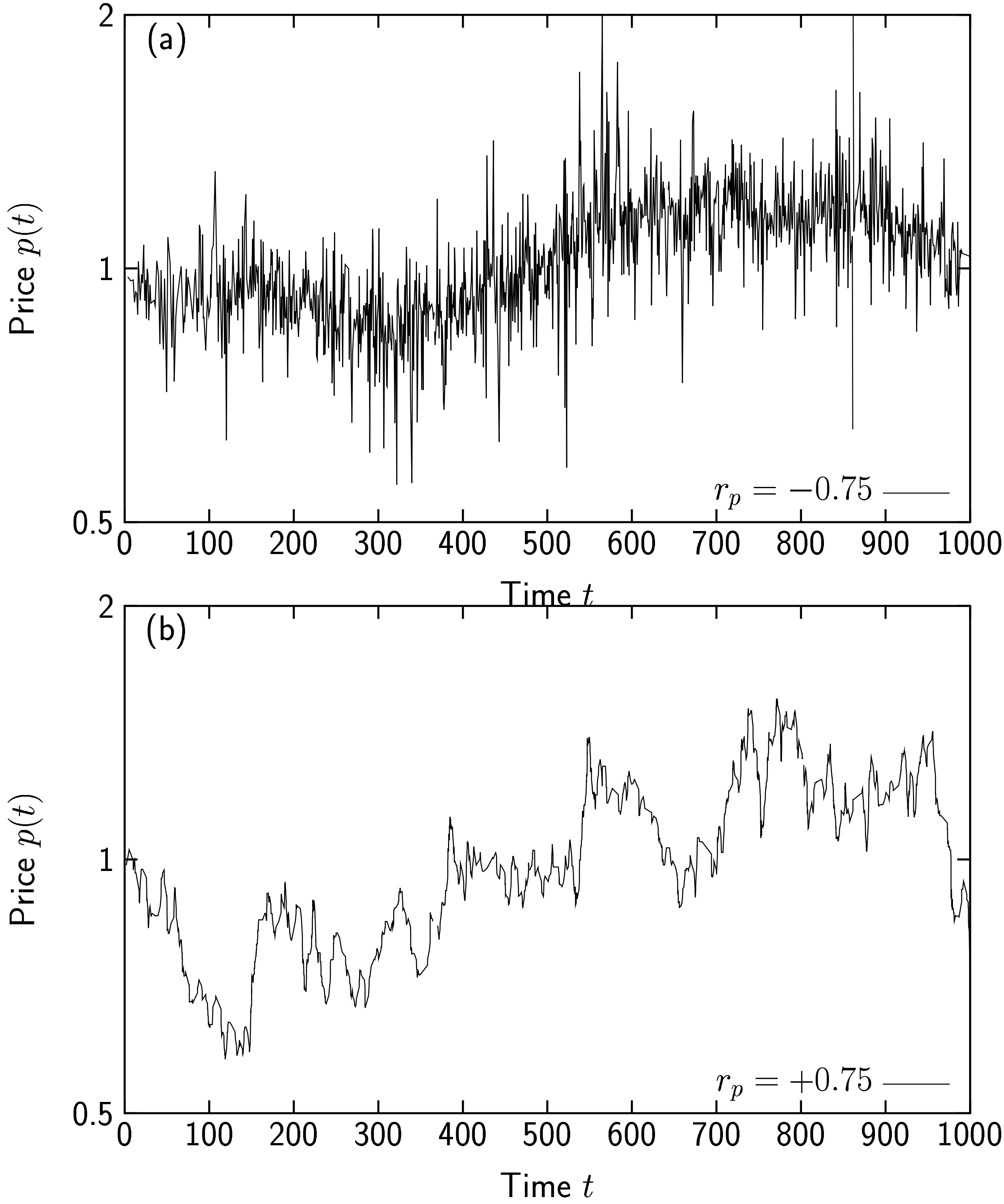


So, as $\sigma_\epsilon \to \sigma_c$ correlations between agents build up such that the dynamics destabilize. Correlation length (number of agents) grows as

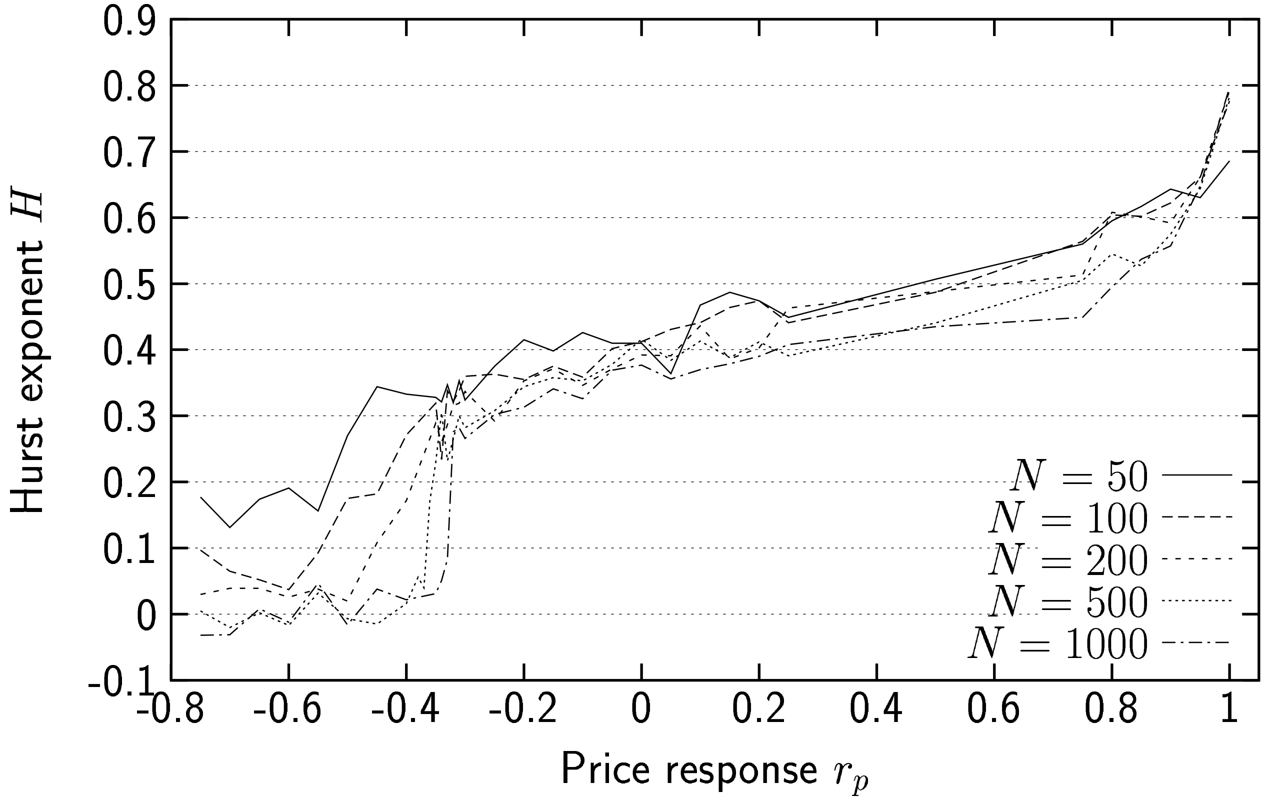
$$\xi \propto (\sigma_\epsilon - \sigma_c)^{-\nu} \tag{10}$$

with $\nu = 1.8 \pm 0.1$. (Would be interesting to know if this is an established universality class.)

5.2 DSEM



In DSEM interesting parameter is price response r_p which sets memory.



Two phase transitions:

1) Goes to $H = 1$ for $r_p \geq 1$. Critical point. (Correlations spanning all agents.)

Fitting to power law

$$1 - H(r_p) \propto (1 - r_p)^b \tag{11}$$

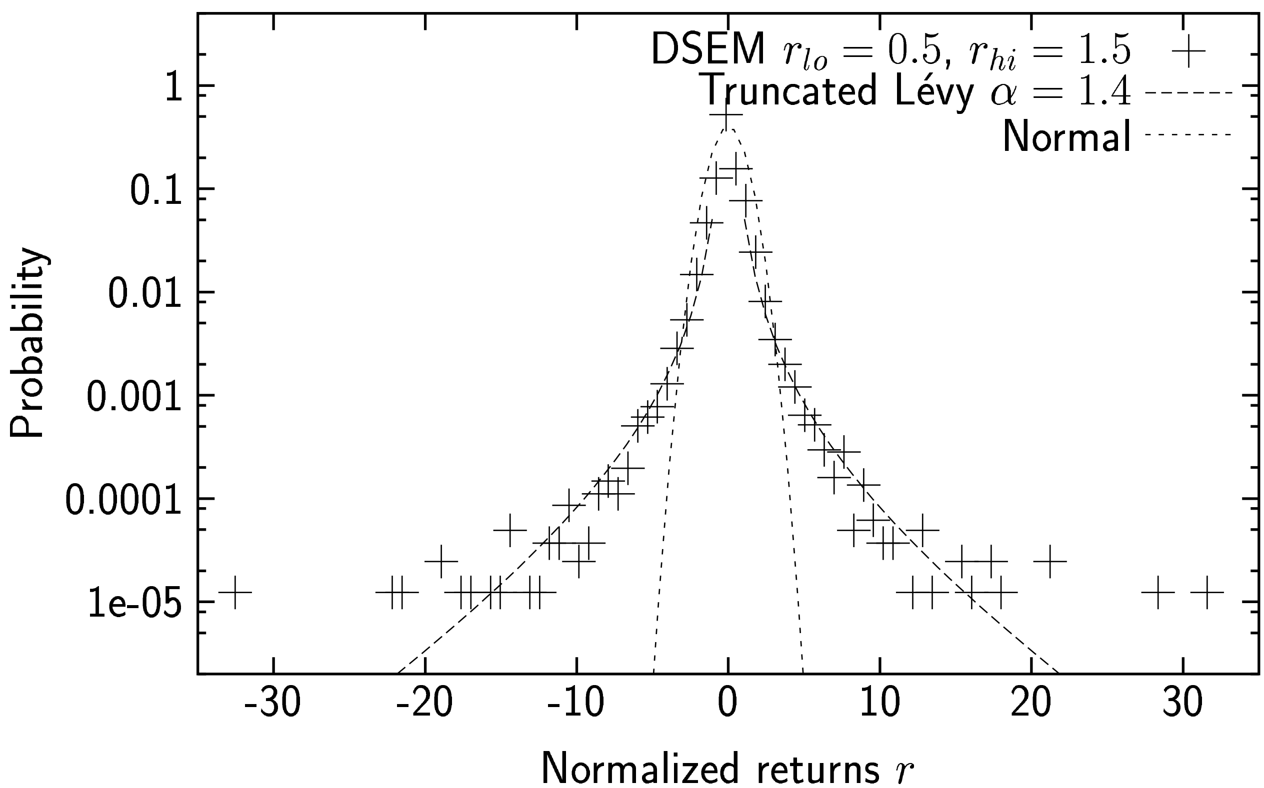
gives exponent $b = 0.19 \pm 0.02$.

2) Goes to $H \approx 0$ for $r_p < -0.33$. First order transition. (Nucleation, intermittency.)

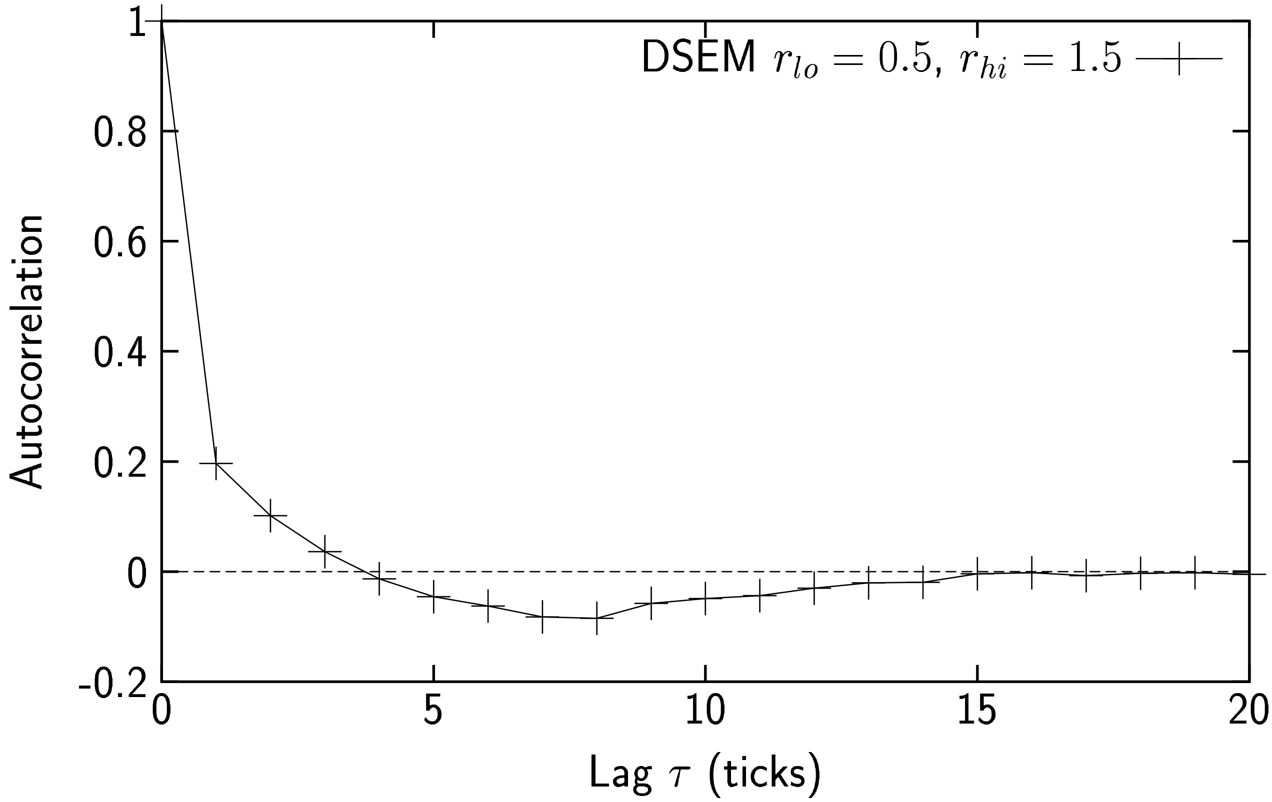
6 How well do they describe real markets?

DSEM was more successful. When control parameter spanned critical point, saw...

Fat tails:

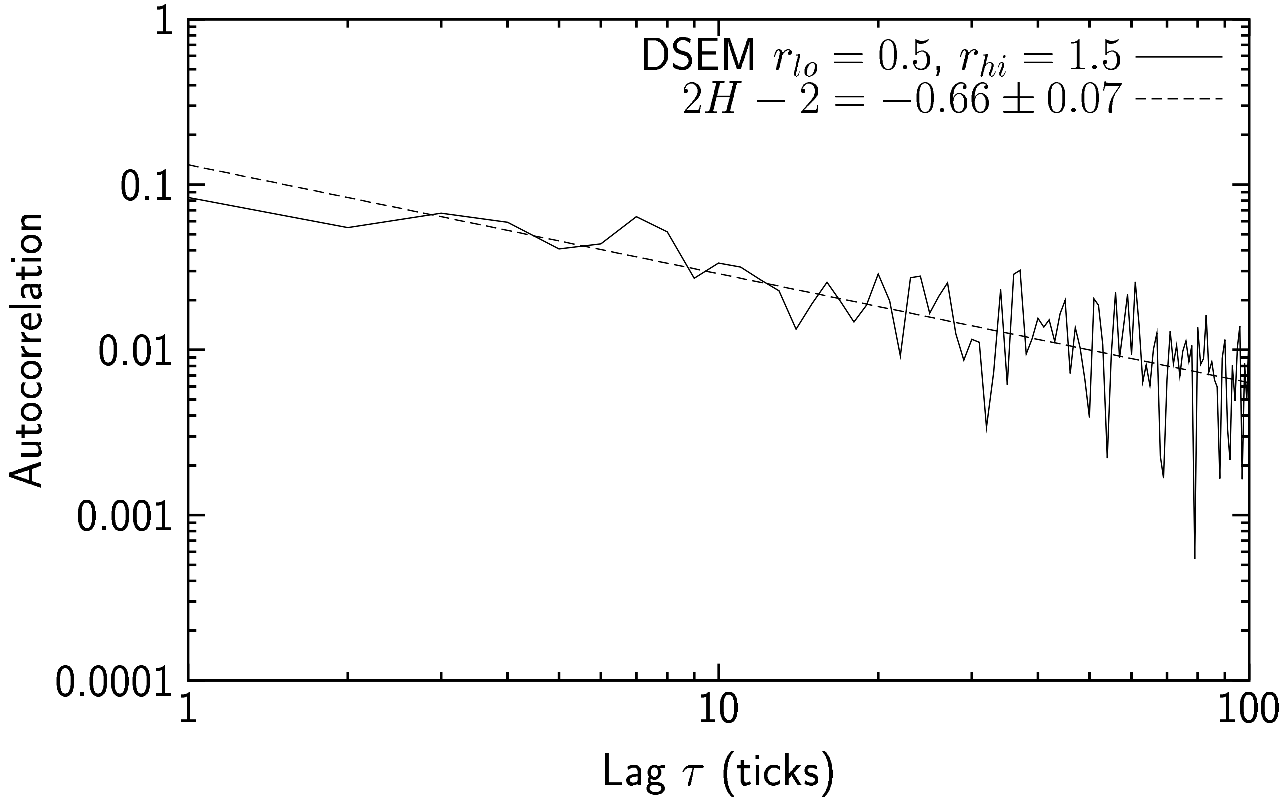


Short memory:



Note timescale is “ticks”—number of trades—not days.

Clustered volatility:



Hurst exponent is $H \approx 0.7$, lower than empirical $H \approx 0.9$ (but still significant).

7 What was all that, again?

Built two models to explain market dynamics.

Both contain critical points.

Both are best able to replicate observed properties when control parameter spans critical point. (DSEM is quite good.)

Only other model which exhibits these properties, Cont-Bouchaud herding model [6], also is near critical point.

8 What's next?

Models suggest empirical markets may be near critical point. If so, why do they self-organize toward criticality (SOC)?

9 Where can I find out more?

My thesis will soon be available from
`http://rikblok.cjb.net/phd` (for you insomniacs).

You can get the simulations described here (source code and 32-bit MS-Windows executables) from
`http://rikblok.cjb.net/csem` and
`http://rikblok.cjb.net/dsem`.

Or check out these...

References

[1] B. B. Mandelbrot, J. Business **36**, 394 (1963).

[2] R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995).

[3] Dow Jones Industrial Average: Daily close, 1896–1999, available from `http://www.economagic.com/em-cgi/data.exe/djind/day-djiac`, provided by `Economagic.com`.

[4] R. C. Merton, *Continuous-Time Finance* (Blackwell, Cambridge, 1992).

[5] E. T. Jaynes, `ftp://bayes.wustl.edu/pub/Jaynes/book.probability.theory/` (unpublished).

[6] R. Cont and J.-P. Bouchaud, cond-mat/9712318 (unpublished).