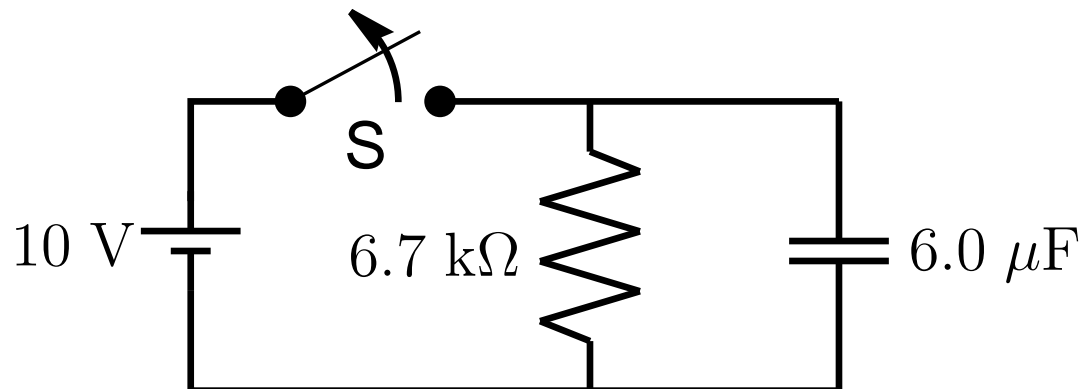


# Tutorial 10 Question

- After the switch  $S$  has been closed for a long time in the circuit shown, it is opened. (a) What is the charge on the capacitor just after the switch is opened? (b) Does the current flow clockwise or counter-clockwise? (c) How long does it take the capacitor to discharge to  $0.1\text{ V}$ ? (d) What is the voltage across the resistor at this time? (e) There is something missing in this circuit. What is it and why does it need to be there? (Hint: what happened when the switch was first closed?)



# Solution

(a) What is the charge on the capacitor just after the switch is opened?

- If the switch has been closed a long time we can assume the voltage across the capacitor (or resistor) has reached 10 V. So the charge on the plates is

$$\begin{aligned} Q &= CV = (6.0 \mu\text{F})(10 \text{ V}) \\ &= 60 \mu\text{C}. \end{aligned}$$

(b) Does the current flow clockwise or counter-clockwise?

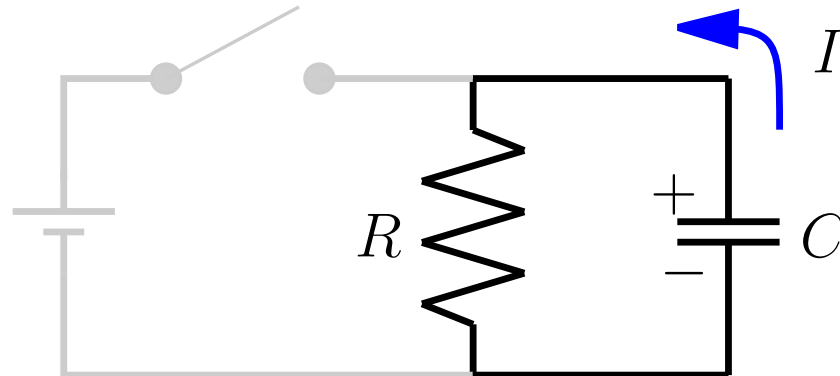
- When the switch was closed a positive charge built up on the upper plate of the capacitor and a negative charge on the lower plate.
- When the switch is opened there is no longer a voltage holding those charges in place.



# Solution, contd

(b) contd

- So the current flows from the upper plate to the lower along the wire—in a **counterclockwise** direction.



(c) How long does it take the capacitor to discharge to 0.1 V?

- We can now ignore the left loop in the circuit because the switch is open so we are just dealing with a capacitor and resistor in series.

# Solution, contd

(c) contd

- In that case the time constant  $\tau$  is just

$$\tau = RC = (6.7 \text{ k}\Omega)(6.0 \text{ }\mu\text{F}) = 40 \text{ ms.}$$

- In the steady state all the charge will leak off the capacitor so  $V_\infty = \frac{Q_\infty}{C} = 0$ . So the voltage across the capacitor follows

$$V(t) - V_\infty = (V_0 - V_\infty) e^{-t/\tau}$$

$$V(t) - 0 = (10 \text{ V} - 0) e^{-t/(40 \text{ ms})}$$

$$V(t) = (10 \text{ V}) e^{-t/(40 \text{ ms})}.$$



# Solution, contd

(c) contd

- We just need to solve  $V(t) = 0.1 \text{ V}$  for  $t$ , which has the solution

$$t = -\tau \ln \frac{V(t)}{V_0}$$

$$t = -(40 \text{ ms}) \ln \frac{0.1 \text{ V}}{10 \text{ V}}$$

$$t = 190 \text{ ms.}$$

- The voltage across the capacitor reaches 0.1 V 190 ms after the switch is opened.

(d) What is the voltage across the resistor at this time?

- By Kirchhoff's loop rule, the voltage across the resistor must also be 0.1 V.



# Solution, contd

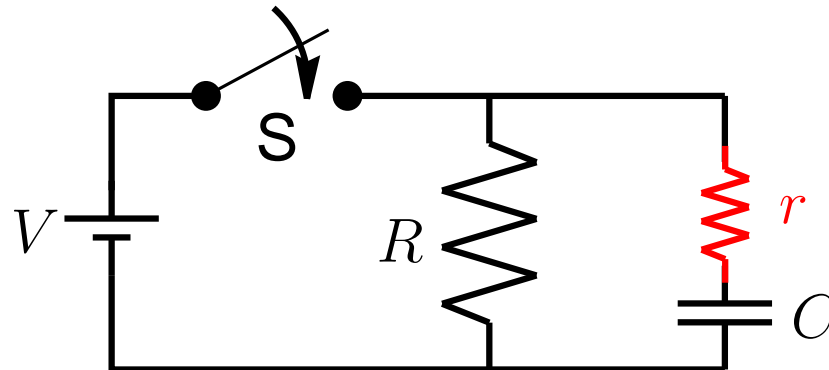
- (e) There is something missing in this circuit. What is it and why does it need to be there? (Hint: what happened when the switch was first closed?)
- When the switch was first closed it appears the voltage across the capacitor was immediately 10 V (from Kirchhoff's loop rule).
  - But that doesn't make sense because it takes time for a charge (and hence, voltage) to build up on the capacitor.
  - So the voltage drop around that part of the circuit must have been due to something else.
  - The something else could have been the resistance of the wire (which we normally take to be negligible but can't in this case).



# Solution, contd

(e) contd

- To represent the wire resistance we should insert a resistor  $r$  in the path of the capacitor:



- As long as  $r \ll R$  we can neglect it when talking about what happens after we open the switch.
- But  $r$  is important in determining how long the capacitor took to charge when the switch was first closed.

