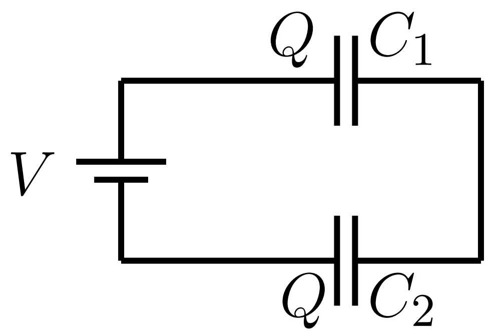
Tutorial 8 Question

- Text: Ch. 24: Pr. 36.
- Two capacitors, $C_1 = 3200\ \mathrm{pF}$ and $C_2 = 2200\ \mathrm{pF}$, are connected in series to a $12.0\ \mathrm{V}$ battery. The capacitors are later disconnected from the battery and connected directly to each other, positive plate to positive plate, and negative plate to negative plate. What then will be the charge on each capacitor?
- Hint: Remember, charge is conserved.

Solution



- First we need to know how much charge is placed on the capacitors. Since they are in series they must both acquire the same charge, Q.

Solution, contd

- To find Q we replace the combination by a single capacitor with the equivalent capacitance,

$$\begin{aligned} C_{eq} &= \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} \\ &= \left[\frac{1}{3200\ \mathrm{pF}} + \frac{1}{2200\ \mathrm{pF}}\right]^{-1} \\ &= 1300\ \mathrm{pF}. \end{aligned}$$

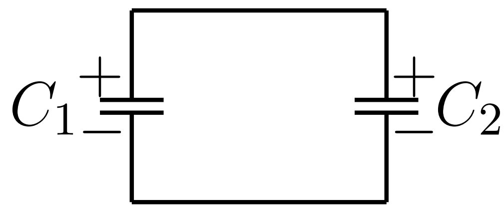
- This capacitor would acquire a charge

$$\begin{aligned} Q &= C_{eq}V = (1300\ \mathrm{pF})(12.0\ \mathrm{V}) \\ &= 15.6\ \mathrm{nC}. \end{aligned}$$

- So that is the charge initially on each capacitor.

Solution, contd

- Now we disconnect the circuit and reconnect the capacitors, $+Q$ to $+Q$, and $-Q$ to $-Q$, as shown below.



- Initially, each plate still has charge $\pm Q$ on it but, since the capacitors aren't equal, there will be a rebalancing of the charge.
- The charges are constrained to remain on their respective sides of the circuit, the $+$ charges can't jump across the gap to the $-$ and vice versa.

Solution, contd

- So, no matter how the charge is redistributed, the total charge on each side will be

$$Q_{\pm} = \pm 2Q.$$

- If we call the charges on the plates Q_1 and Q_2 then on the $+$ side we have

$$Q_{+} = 2Q = Q_1 + Q_2.$$

- (Looking at Q_{-} would give us the exact same relationship.)
- The charges will flow until the voltages on the two capacitors exactly cancel each other (the sum of V over a loop is zero). So, eventually $V_1 = V_2$.

Solution, contd

- Then, from $\boxed{Q = CV}$ we have

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}.$$

- So we have two equations in two unknowns (Q_1 and Q_2). After a bit of work we find

$$\begin{aligned} Q_1 &= \frac{2QC_1}{C_1 + C_2} = 18.5\ \mathrm{nC}, \\ Q_2 &= \frac{2QC_2}{C_1 + C_2} = 12.7\ \mathrm{nC}. \end{aligned}$$

- These are the final charges on the capacitors. □