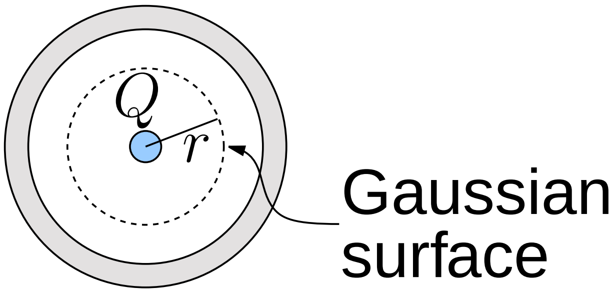
Tutorial 6 Question

- Text: Ch. 22: Pr. 16.
- A point charge Q rests at the center of an uncharged thin spherical conducting shell. What is the electric field E as a function of r (*a*) for r less that the radius of the shell, (*b*) inside the shell, and (*c*) beyond the shell? (*d*) Does the shell affect the field due to Q alone? Does the charge Q affect the shell?

Solution

(a) What is the electric field E as a function of r for r less that the radius of the shell?

- From spherical symmetry, electric field must point radially away/to charge.
- So good choice of Gaussian surface is sphere.
- Then electric field is $\perp$ to surface everywhere.



- Enclosed charge: $Q_{\text{encl}} = Q$.
- Area: $A = 4\pi r^2$ (surface area of sphere).

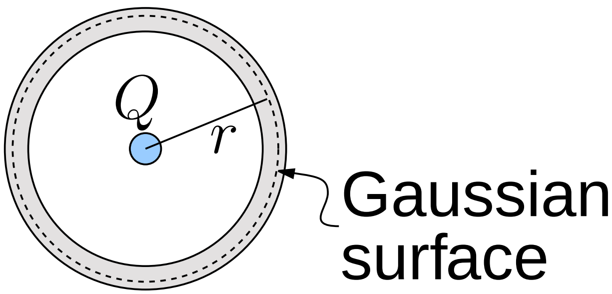
Solution, contd

- Flux: $\Phi_E = E_\perp A = E(4\pi r^2)$.
- Gauss's law: $E_\perp A = Q_{\text{encl}}/\epsilon_0$, so

$$E = \frac{Q}{4\pi\epsilon_0\, r^2}.$$

(*b*) What is the electric field E as a function of r inside the shell?

- Now we expand our Gaussian surface so it lies within the conductor (same symmetry).



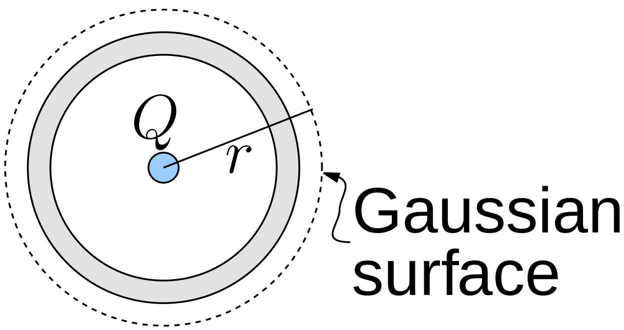
Solution, contd

- Recall, E inside a conductor is always zero,

$$E = 0.$$

(*c*) What is the electric field E as a function of r beyond the shell?

- Again we expand the Gaussian surface so now it is outside the shell.



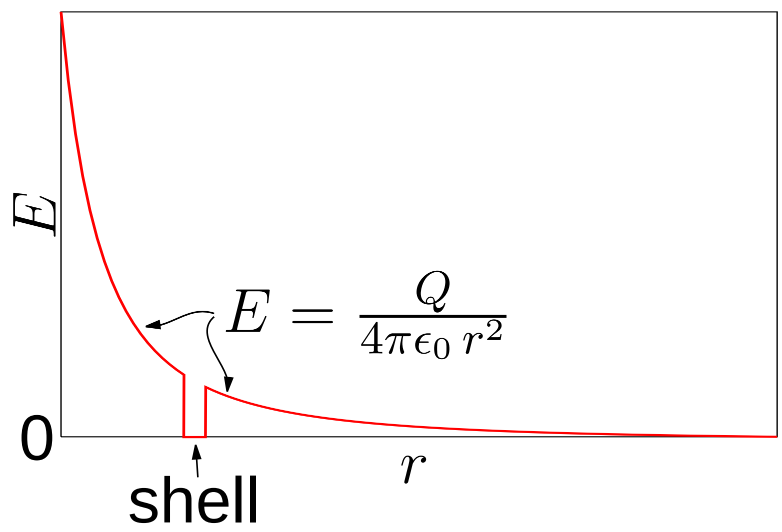
- The shell carries no net charge so $Q_{\text{encl}} = Q$, as before.

Solution, contd

- The surface area of the sphere is still $A = 4\pi r^2$.
- So everything is the same as in part (*a*) and

$$E = \frac{Q}{4\pi\epsilon_0\, r^2}.$$

- Overview: plotting E as a function of radius r...



Solution, contd

(d) Does the shell affect the field due to Q alone?

- Yes, but only within the shell, where $E = 0$. Everywhere else, the field is the same as it would be due to Q alone.

(d) Does the charge Q affect the shell?

- Yes. Without the charge Q the shell would be neutral. But let's think about what happens with Q present.
- As stated in part (b) the electric field inside a conductor is zero. So the Gaussian surface has no flux through it.
- Then the surface must enclose no net charge. So the inner surface of the conductor must have a charge $-Q$ distributed over it to exactly cancel the center charge.

Solution, contd

- To balance that, the outer surface of the conductor must have a charge $+Q$ distributed over it (so that the net enclosed charge in part (*c*) is $+Q$).
- So the center charge affects the shell by attracting a charge $-Q$ to the inner surface and repelling a charge $+Q$ to the outer surface. □