UBC Physics 102

Lecture 15

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Outline

- ▷ AC source
- ▷ R circuits
- ▷ L circuits
- ▷ C circuits
- ▷ LRC circuits
- ▷ Resonance
- ▷ End

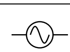
AC source [Text: Sect. 31-1]

- **Definition:** *AC source*
  - Generates an alternating (sinusoidal unless otherwise stated) current,

$$I = I_0 \sin \omega t.$$

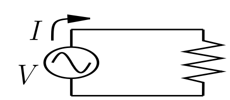
  - Angular frequency ω related to frequency f and period T by

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

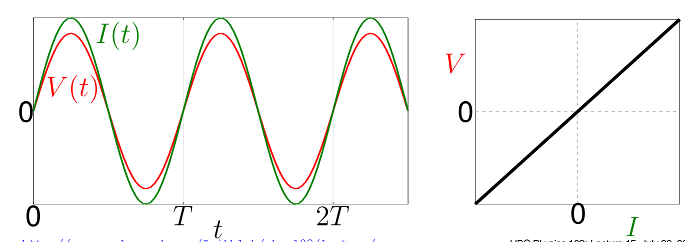
  - Circuit symbol: 
  - Note: multimeters read RMS values, not peak.

R circuits [Text: Sect. 31-2]

- **Discussion: R circuits**

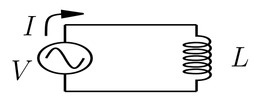


  - Voltage drop across resistor, $V = IR = I_0 R \sin \omega t$ so voltage amplitude is $V_0 = I_0 R$.



L circuits [Text: Sect. 31-3]

- **Discussion: R circuits, contd**
  - RMS voltage, $V_{\mathrm{RMS}} = I_{\mathrm{RMS}} R$.
- **Interactive Quiz: PRS 15a**
- **Discussion: L circuits**

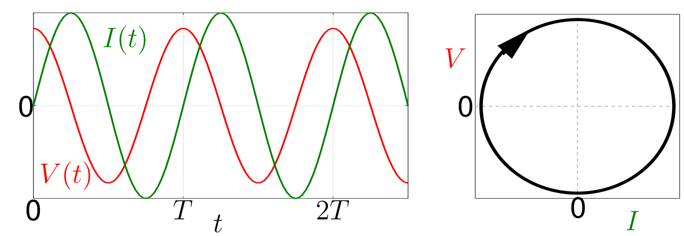


  - By Kirchhoff's loop rule,

$$V = L\frac{dI}{dt} = L\frac{d}{dt} I_0 \sin\omega t$$
$$= \omega L I_0 \cos\omega t.$$

L circuits, contd

- **Discussion: L circuits, contd**
  - Current *lags* voltage by $90°$.



L circuits, contd

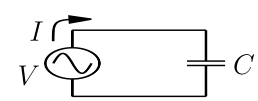
- **Definition:** *Inductive reactance,* X_L
  - Voltage amplitude is $V_0 = \omega L I_0$.
  - Multimeter will give readings $V_{\mathrm{RMS}} = I_{\mathrm{RMS}} X_L$ where

$$X_L = \omega L.$$

  - X_L called inductive *reactance*.
  - Like resistance except current out of phase with voltage.

C circuits [Text: Sect. 31-4]

- **Discussion: C circuits**



  - By Kirchhoff's loop rule, $V = \frac{Q}{C}$.
  - How does this relate to current? $I = \frac{dQ}{dt}$ so

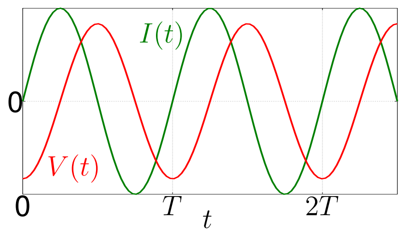
$$\frac{dV}{dt} = \frac{I}{C} = \frac{I_0}{C} \sin\omega t.$$

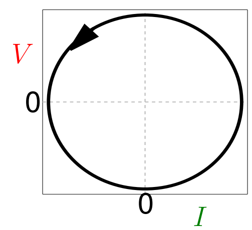
  - Solution is

$$V = -\frac{I_0}{\omega C} \cos\omega t.$$

C circuits, contd

- **Discussion: C circuits, contd**
  - Current *leads* voltage by $90°$.





C circuits, contd

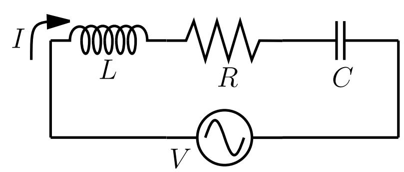
- **Definition:** *Capacitive reactance,* X_C
  - Voltage amplitude is $V_0 = \frac{I_0}{\omega C}$.
  - Multimeter will give readings $V_{\rm RMS} = I_{\rm RMS} X_C$ where

$$X_C = \frac{1}{\omega C}.$$

  - X_C called inductive *reactance*.
- **Discussion: CIVIL memory aid**
  - **CIV**IL = in **C**apacitors **I** comes before **V**.
  - CI**VIL** = **V** comes before **I** in inductors (**L**).

LRC circuits [Text: Sect. 31-5]

- **Discussion: LRC circuits**



  - What if circuit contains all three components in series?
  - Same current $I = I_0 \sin\omega t$ goes through each.
  - Expect total voltage drop to be $V = V_0 \sin(\omega t + \phi)$.
  - But which ϕ? $0°$, $\pm 90°$ or something else?
  - And what is V_0?

LRC circuits, contd

- **Discussion: LRC circuits, contd**
  - From Kirchhoff's loop rule,

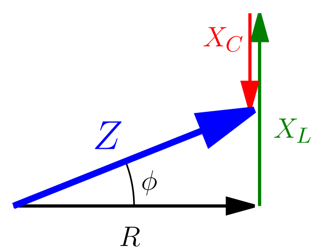
$$\begin{aligned} V &= V_R + V_L + V_C \\ &= I_0 R \sin\omega t + I_0 (X_L - X_C) \cos\omega t. \end{aligned}$$

- **Definition:** *Impedance*
  - After some geometry (hand waving) we find $V_0 = I_0 Z$.
  - Likewise, multimeters will read $V_{\rm RMS} = I_{\rm RMS} Z$ where

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

LRC circuits, contd

- **Definition:** *Impedance, contd*
  - Z called impedance of circuit.
  - "Impedes" current flow like resistance but also phase-shifts it by ϕ.
  - Vector representation:



LRC circuits, contd

- **Definition:** *Impedance, contd*
  - Summary:

Name	Symbol	Phase shift
Resistance	R	$0°$
Reactance	X	$\pm 90°$
Impedance	Z	$\phi \ (\lvert\phi\rvert \leq 90°)$

- **Interactive Quiz: PRS 15b**
- **Derivation: Power**
  - L and C don't lose energy, they just swap it back and forth.
  - Only R loses energy, as heat.

LRC circuits, contd

- **Derivation: Power, contd**
  - Average rate of power consumption is $\boxed{\overline{P} = I_{\text{RMS}}^2 R}$.
  - From vector representation we see $R = Z\cos\phi$ so

$$\boxed{\overline{P} = I_{\text{RMS}}^2 Z \cos\phi = I_{\text{RMS}} V_{\text{RMS}} \cos\phi.}$$

  - $\cos\phi$ called **power factor**.

Resonance [Text: Sect. 31-6]

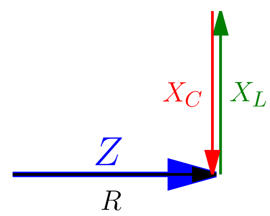
- **Discussion: Resonance**
  - Already saw a LC circuit (without any voltage source) oscillates at frequency $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$.
  - What is so special about that frequency?
  - If we "push" an LRC circuit with an AC source at frequency ω_0,

$$X_L = \omega_0 L = \sqrt{\frac{L}{C}},$$
$$X_C = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}.$$

  - So $X_L = X_C$.

Resonance, contd

- **Discussion: Resonance, contd**



  - So $\phi = 0$ and $Z = R$. Changing ω can only make Z bigger.
- **Interactive Quiz: PRS 15c**

End

- **Practice Problems:**
  - Ch. 31: Q. 1, 3, 5, 7, 9, 11, 13, 15.
  - Ch. 31: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 35, 37, 39, 41, 43, 45, 51, 53.
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut15**