UBC Physics 102

Lecture 14

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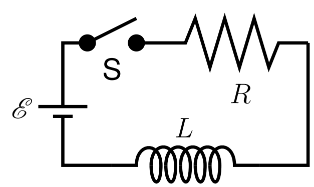
Outline

- ▷ LR circuits
- ▷ LRC circuits
- ▷ End

LR circuits [Text: Sect. 30-4]

- **Discussion: LR circuits**



- When switch S closed, current gradually increases from zero.
- When current still small, little voltage lost in resistor, $V_R = IR \approx 0$.
- So most voltage lost in inductor, $V_L \approx \mathscr{E}$ (Kirchhoff's loop rule).

LR circuits, contd

- **Discussion: LR circuits, contd**
  - So V_L starts at maximum and falls off to zero.
  - Eventually $V_L \to V_\infty = 0$ and inductor just acts like regular wire (can be ignored).
  - Current changes from initial I_0 to final I_∞ exponentially,

$$\begin{aligned} V_L - V_\infty &= (V_0 - V_\infty)e^{-t/\tau}, \\ I - I_\infty &= (I_0 - I_\infty)e^{-t/\tau}. \end{aligned}$$

  - In this case $I_0 = 0$ but not always.

LR circuits, contd

- **Discussion: LR circuits, contd**
  - Time constant (time to get $\sim 2/3$ of the way) is

$$\boxed{\tau = \frac{L}{R}.}$$

  - Time constant only applies to simple circuit with one L and R in series.
  - So can determine behaviour by just finding values of V_0, I_0, and I_∞.
  - Similar to RC circuits.
- **Interactive Quiz: PRS 14a**

LR circuits, contd

- **Example: Pr. 54**
  - Two tightly wound solenoids have the same length and circular cross-sectional area. But solenoid 1 uses wire that is half as thick as solenoid 2. (*a*) What is the ratio of their inductances? (*b*) What is the ratio of their inductive time constants?
- **Solution: Pr. 54**

(*a*) What is the ratio of their inductances?

  - If the wire 2 is twice as thick then it can only fit half as many turns of the wire over the same length so $N_1 = 2N_2$.

LR circuits, contd

- **Solution: Pr. 54, contd**
  - The inductance of a solenoid is $\boxed{L = \frac{\mu_0 N^2 A}{l}}$ and both are the same in every respect expect N so

$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} = \frac{2^2}{1^2} = 4.$$

(*b*) What is the ratio of their inductive time constants?

  - The resistance of the wire in a solenoid is $\boxed{R = \rho \frac{l_w}{A_w}}$ where l_w is now the total length of wire and A_w is the cross-sectional area of the wire itself.
  - Again, because we can only fit half as many turns on coil 2, $l_{w1} = 2l_{w2}$.

LR circuits, contd

- **Solution: Pr. 54, contd**
  - If wire 2 is twice as thick then it has $A_{w2} = 4A_{w1}$ the cross-sectional area.
  - So the ratio of the wires' resistances is

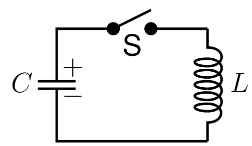
$$\frac{R_1}{R_2} = \left(\frac{l_{w1}}{l_{w2}}\right)\left(\frac{A_{w2}}{A_{w1}}\right) = (2)(4) = 8.$$

  - Finally, the time constant is $\boxed{\tau = L/R}$ so

$$\frac{\tau_1}{\tau_2} = \left(\frac{L_1}{L_2}\right)\left(\frac{R_2}{R_1}\right) = (4)\left(\frac{1}{8}\right) = \frac{1}{2}. \quad \square$$

LRC circuits [Text: Sect. 30-5,6]

- **Derivation: LC circuits**



  - Can construct circuit with just inductor and capacitor.
  - What happens if C initially charged when S closed?
  - Kirchhoff's loop rule:

$$V_C + V_L = 0 = \frac{Q}{C} - L\frac{dI}{dt}.$$

LRC circuits, contd

- **Derivation: LC circuits, contd**
  - Charge lost off capacitor $-\frac{dQ}{dt}$ generates current I,

$$I = -\frac{dQ}{dt}.$$

  - So charge related to its own second derivative,

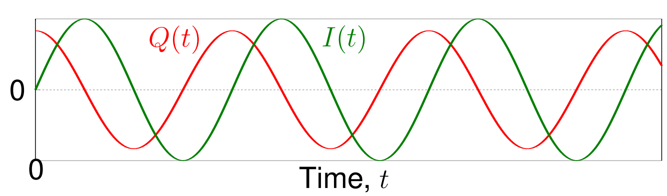
$$\frac{Q}{C} = -L\frac{d^2Q}{dt^2}.$$

  - Solution is oscillation, $Q(t) = Q_0 \cos(\omega_0 t)$ where

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

LRC circuits, contd

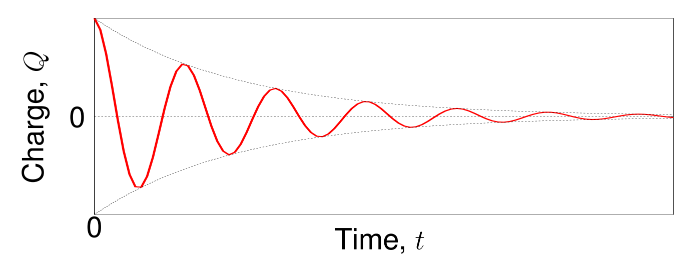
- **Derivation: LC circuits, contd**
  - Charge and current both oscillate.
  - Frequency ω_0 determined by L and C.
  - $I = -\frac{dQ}{dt}$ so current lags charge.



  - Stored energy oscillates between capacitor and inductor.
- **Interactive Quiz: PRS 14b**

LRC circuits, contd

- **Discussion: LRC circuits**
  - If circuit contains resistance then power lost in each cycle.
  - So amplitude decreases (damped oscillations).



  - Can supply oscillating power to compensate for loss.

End

- **Practice Problems:**
  - Ch. 30: Q. 9, 11, 13, 15, 17, 19.
  - Ch. 30: Pr. 7, 13, 15, 17, 25, 27, 29, 31, 33, 35, 45.
- **Interactive Quiz: Feedback**