UBC Physics 102

Lecture 12

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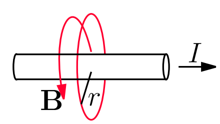
Outline

Straight wire [Text: Sect. 28-1]

- **Discussion: Straight wire**
  - Magnetic field due to current in a long straight wire.



  - Stronger closer to wire, $B \propto \frac{1}{r}$, and with stronger current, $B \propto I$.
  - Will derive later that

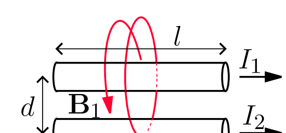
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}.$$

Force between wires [Text: Sect. 28-2]

- **Definition:** *Permeability of free space,* μ_0

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}.$$

- **Discussion: Force between wires**
  - Already saw B-field produces force on wire.
  - If wires also produce B-fields then 2 parallel wires will have force on each other.



Force between wires, contd

- **Discussion: Force between wires, contd**
  - B-field due to wire 1 at distance d is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

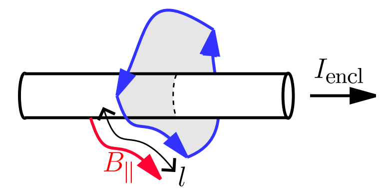
  - Force on wire 2 in B_1 given by $F_{2/1} = I_2 l B$ so force is

$$F_{2/1} = \frac{\mu_0 I_1 I_2}{2\pi d} l.$$

  - Increases with length l.
- **Interactive Quiz: PRS 12a**

Ampère's law [Text: Sect. 28-4]

- **Definition:** *Ampère's law*



  - If a current I_{encl} passes through a closed loop then

$$\sum_{\text{segments}} B_{||} l = \mu_0 I_{encl}.$$

Ampère's law, contd

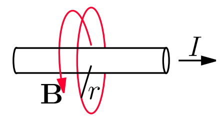
- **Definition:** *Ampère's law, contd*
  - I_{encl} is sum of of all current going through loop in same direction (subtract if reversed).
  - $\sum_{\text{segments}}$ is sum over all segments of loop.
  - l is length of segment.
  - $B_{||}$ is field parallel to segment.
  - Use right-hand field rule to choose direction of path.
- **Discussion: Ampère's law**
  - Parallels Gauss's law but deals with loops instead of surfaces.
  - Second of Maxwell's 4 equations.

Ampère's law, contd

- **Discussion: Ampère's law, contd**
  - You get to choose "Amperian" loop. Use symmetry.
  - Want $\mathbf{B}$ $\perp$ or $\|$ to each segment.
  - $\perp$ segments can be dropped.
- **Derivation: Long, straight wire**
  - Infinitely long straight wire.
  - From Right-hand field rule B-field wraps around wire.
  - From symmetry must be a circle (has to look the same no matter how you rotate the system).
  - So we pick circular Amperian loop (1 continuous segment).

Ampère's law, contd

- **Derivation: Long, straight wire, contd**
  - $B_{\parallel} = B$ everywhere on circle.



  - Length of segment (circumference) is $l = 2\pi r$.
  - Enclosed current is just $I_{\rm encl} = I$.
  - Ampère's law:

$$\sum_{\rm segments} B_{\parallel} l = \mu_0 I_{\rm encl}$$
$$B(2\pi r) = \mu_0 I.$$

Ampère's law, contd

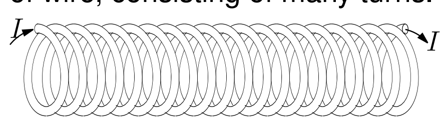
- **Derivation: Long, straight wire, contd**
  - So we find

$$B = \frac{\mu_0 I}{2\pi r}.$$

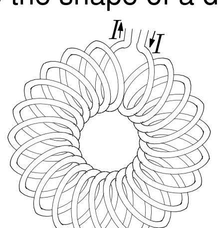
  - Is magnetic field around a long, straight wire. □
- **Interactive Quiz: PRS 12b**

Solenoids and toroids [Text: Sect. 28-5]

- **Definition:** ***Solenoid***
  - Long coil of wire, consisting of many turns.



- **Definition:** ***Toroid***
  - Solenoid bent into the shape of a donut (torus).



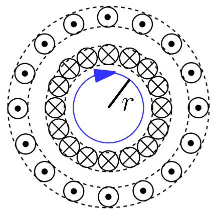
Solenoids and toroids, contd

- **Principle: Superposition**
  - Like E-field, can find net B-field by adding up B's due to each wire.
- **Derivation: Toroid magnetic field**
  - Can use Ampère's law to find B-field in/around toroid.
  - By symmetry loop should be circle of radius r.
  - 3 cases: (1) loop smaller than toroid, (2) loop inside toroid, (3) loop bigger than toroid.

Solenoids and toroids, contd

- **Derivation: Toroid magnetic field, contd**

Case 1: Cross-sectional view of toroid:



$\odot = I$ out
$\otimes = I$ in

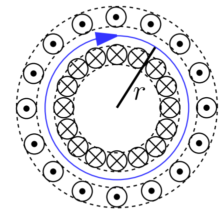
- $I_{\text{encl}} = 0$ and $B = B_{\parallel}$ so for any r we find

$$B = 0.$$

Solenoids and toroids, contd

- **Derivation: Toroid magnetic field, contd**

Case 2: Cross-section:



$\odot = I$ out
$\otimes = I$ in

- If there are N turns then $I_{\text{encl}} = NI$ so

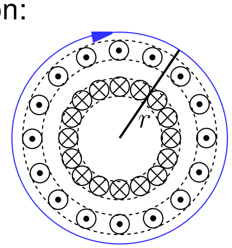
$$B = \mu_0 \frac{N}{l} I.$$

- ($l = 2\pi r$ but it's handy to leave it as l.)

Solenoids and toroids, contd

- **Derivation: Toroid magnetic field, contd**

Case 3: Cross-section:



$\odot = I$ out
$\otimes = I$ in

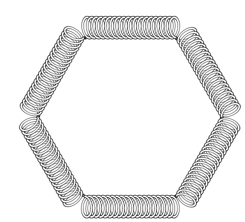
- Again, $I_{\text{encl}} = 0$ (they all cancel) so

$$B = 0.$$

- So $B = 0$ everywhere outside toroid and $B = \mu_0 \frac{N}{l} I$ inside. □

Solenoids and toroids, contd

- **Derivation: Solenoid magnetic field**
  - We can construct a toroid from many solenoids laid in a circle.



  - So each solenoid must have same field,

$$B = \mu_0 \frac{N}{l} I.$$

Solenoids and toroids, contd

- **Derivation: Solenoid magnetic field, contd**
  - And $B = 0$ (roughly) outside solenoid.
  - $\frac{N}{l}$ is # turns per unit length, often written n (eg. $B = \mu_0 n I$).
  - Use RH field rule to determine direction.
- **Discussion: Solenoid**
  - B very uniform inside solenoid (far from ends).
  - B gets weaker and starts to spread near ends.
  - Behaves like bar magnet (B comes out of N, goes into S end.)
- **Interactive Quiz: PRS 12c**

End

- **Practice Problems:**
  - Ch. 28: Q. 1, 3, 5, 7, 9, 11, 21, 23.
  - Ch. 28: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23 ,25, 27, 47, 49, 55, 59, 61, 63.
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut12**