UBC Physics 102

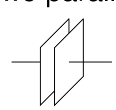
Lecture 8

Rik Blok

Outline

Capacitors [Text: Sect. 24-1]

- **Definition:** *Capacitor*
  - Device that can store electric charge.
  - Often constructed of two parallel plates.



  - Circuit diagram symbol: ⊣⊢
  - When voltage V applied to plates they acquire charges $\pm Q$ where

$$Q = CV.$$

Capacitance [Text: Sect. 24-2]

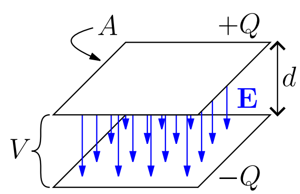
- **Definition:** *Capacitance, C*
  - Constant of proportionality.
  - Depends on details of capacitor (eg. size, shape).
- **Unit: Farad, F**
  - Unit of capacitance,

$$1\ \mathrm{F} = 1\ \mathrm{C/V}.$$

- **Discussion: Capacitance**
  - To derive capacitance solve for relationship between V and Q. Will find $Q \propto V$. C is proportionality constant.

Capacitance, contd

- **Derivation: Parallel plate capacitance**



  - From Gauss's law can find strength of E-field between plates,
  $$E = \frac{\sigma}{\epsilon_0}.$$
  - σ is surface charge density, $\sigma = \frac{Q}{A}$.

Capacitance, contd

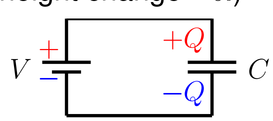
- **Derivation: Parallel plate capacitance, contd**
  - E uniform so can use $\boxed{V = -E_l l}$ (ignoring sign) to get
  $$V = \frac{\sigma d}{\epsilon_0} = \frac{Qd}{\epsilon_0 A}.$$
  - Rearranging gives $Q = \frac{\epsilon_0 A}{d} V = CV$ so

$$\boxed{C = \frac{\epsilon_0 A}{d}.}$$

  - Can use similar method for other shapes.

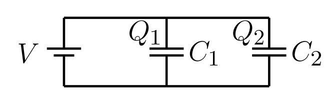
Series and parallel [Text: Sect. 24-3]

- **Discussion: Review of circuit diagrams**
  - Lines are wires connecting components. No voltage drop.
  - Battery (Symbol: $^{+}$⊣⊢$^{-}$) raises voltage on $+$ side. (Analogy: an escalator.)
  - Capacitor: ⊣⊢
  - Voltage pumps charge onto plates.
  - Net voltage change over any loop must be zero. (Analogy: net height change$= 0$.)



Series and parallel, contd

- **Definition:** *Parallel*
  - Path splits, goes across multiple components, and rejoins.
  - Voltage drop identical across each path.
- **Derivation: Parallel capacitors**



  - Same voltage drop across each capacitor so $V_1 = V_2 = V$.
  - Charge on capacitors: $Q_1 = C_1 V$, $Q_2 = C_2 V$.

Series and parallel, contd

- **Derivation: Parallel capacitors, contd**
  - Total charge Q pumped onto both capacitors,

$$\begin{aligned} Q &= Q_1 + Q_2 = C_1 V + C_2 V \\ &= (C_1 + C_2) V. \end{aligned}$$

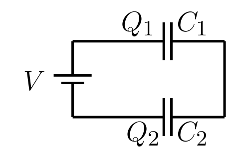
  - Equivalent to a single capacitor, $Q = C_{eq} V$, where

$$C_{eq} = C_1 + C_2.$$

- **Definition:** *Series*
  - Components are connected end to end.
  - Total voltage drop is sum of all components.

Series and parallel, contd

- **Derivation: Series capacitors**



  - Charge conserved so + charge built up on capacitor 1 must leave − charge on capacitor 2, and vice versa.
  - So charge the same on both capacitors, $Q_1 = Q_2 = Q$.
  - Voltage drop across capacitors: $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$.

Series and parallel, contd

- **Derivation: Series capacitors, contd**
  - Total voltage drop across capacitors equal to voltage climb across battery,

$$\begin{aligned} V &= V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \\ &= \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q. \end{aligned}$$

  - Equivalent to a single capacitor, $V = \frac{1}{C_{eq}} Q$, where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Series and parallel, contd

- **Discussion: Series and parallel capacitors**
  - Can use these “replacement” rules to simplify complex configurations of capacitors.
  - Rarely not able to divide circuit into series and parallel. But voltage rules (eg. $= 0$ over loop), charge conservation, and other laws still apply.
- **Interactive Quiz: PRS 08a**

Energy storage [Text: Sect. 24-4]

- **Discussion: Energy storage**
  - Have to do work to charge capacitor.
  - Work stored as potential energy in capacitor.
  - Energy stored in electric field (between plates).
- **Derivation: Potential energy**
  - Can derive how much energy stored in capacitor.
  - Consider situation where voltage $\widehat{V}$ ramped up gradually from $\widehat{V} = 0$ to $\widehat{V} = V$.
  - Charge on plates when ramp voltage is $\widehat{V}$ is $Q = C\widehat{V}$.
  - Recall, energy of charge Q at voltage $\widehat{V}$ is $U = Q\widehat{V}$.
  - But both Q and $\widehat{V}$ changing so can't use $U = QV$.

Energy storage, contd

- **Derivation: Potential energy, contd**
  - But if voltage change small enough ($d\widehat{V} \to 0$) then can assume Q constant over change.
  - Increase in potential energy is

$$dU = Q\,d\widehat{V} = C\widehat{V}\,d\widehat{V}.$$

  - Total potential is sum of increments, $U = \int_0^V C\widehat{V}\,d\widehat{V}$,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}.$$

  - Last expression comes from $Q = CV$.

Dielectrics [Text: Sect. 24-5]

- **Definition:** *Dielectric constant,* K
  - Amount by which material between plates increases capacitance,

$$C = KC_0.$$

  - C_0 is capacitance in vacuum (or air, roughly).
  - $K > 1$ for dielectric.
  - Dielectric works by polarizing, setting up electric field contrary to original.
  - Changing C changes potential energy U capacitor will hold.

Dielectrics, contd

- **Interactive Quiz: PRS 08b**
- **Example: Pr. 75**
  - A parallel-plate capacitor is isolated with a charge $\pm Q$ on each plate. If the separation of the plates is halved and a dielectric (constant K) is inserted in place of air, by what factor does the energy storage change? To what do you attribute the change in stored potential energy?
- **Solution: Pr. 75**
  - We know charge Q held constant.

Dielectrics, contd

- **Solution: Pr. 75, contd**
  - So $\boxed{U = \frac{1}{2}\frac{Q^2}{C}}$ gives potential energy. Initially,

$$U_0 = \frac{1}{2}\frac{Q^2}{C_0}.$$

  - Let's think about this in 2 steps:
  - Step 1: halving the separation.
  - From $\boxed{C = \frac{\epsilon_0 A}{d}}$ this has the effect of doubling the capacitance,

$$C_1 = 2C_0.$$

Dielectrics, contd

- **Solution: Pr. 75, contd**
  - Step 2: inserting the dielectric.
  - From $\boxed{C = KC_0}$ the capacitance is increased by a factor K,

$$C_2 = KC_1 = 2KC_0.$$

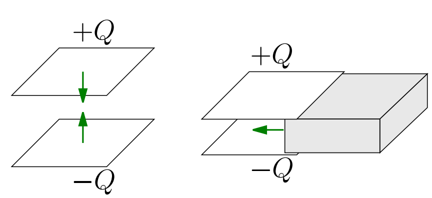
  - Now the potential energy is

$$U_2 = \frac{1}{2}\frac{Q^2}{C_2} = \frac{1}{2K}U_0.$$

  - So potential energy decreases by ratio of $2K$.
  - In each step, the potential energy decreased.
  - So both changes happened spontaneously.

Dielectrics, contd

- **Solution: Pr. 75, contd**



  - Step 1: Releasing plates allowed them to move closer together (attraction of plates).
  - Step 2: Capacitor drew in dielectric when we brought it close (dielectric polarized in proximity of capacitor then attracted to it). □

Electric current [Text: Sect. 25-2]

- **Definition:** *Electric current,* I
  - Rate of charge flow past a point,

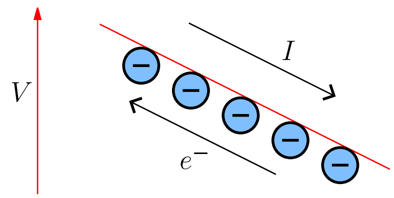
$$I = \frac{dQ}{dt}.$$

  - Current flows if voltage different between points.
  - Analogy: voltage↔height, current↔ river current.
- **Unit: Ampere,** A
  - Unit of current

$$1\ \mathrm{A} = 1\ \mathrm{C/s}.$$

Electric current, contd

- **Discussion: Convention**
  - Current said to flow from high potential to low (historical).
  - Actually, electrons flowing from low to high.



  - Doesn't matter, except in rare cases.

Ohm's law [Text: Sect. 25-3]

- **Definition:** *Ohm's law*
  - Current I proportional to voltage difference V that caused it, $V \propto I$, or

$$V = IR.$$

  - R is proportionality constant, called **resistance**.
  - Water analogy: $\frac{1}{R} \leftrightarrow$ cross-section of pipe (restricts flow).
  - Ohm's law states R is constant.
  - Not always true (will see in lab).

Ohm's law

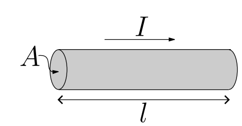
- **Unit: Ohm, Ω**
  - Units of resistance,

$$1\ \Omega = 1\ \mathrm{V/A}.$$

- **Definition:** *Resistor*
  - A circuit component that obeys Ohm's law.
  - Has a specific resistance, R.
  - Circuit diagram symbol: -\/\/\/-
  - R depends on material, size, shape, etc.

Resistivity [Text: Sect. 25-4]

- **Definition:** *Resistivity, ρ*



  - Empirically found that metals obey

$$R = \rho \frac{l}{A}.$$

  - Proportionality constant, ρ, depends on material and temperature, not much else.
- **Interactive Quiz: PRS 08c**

End

- **Practice Problems:**
  - Ch. 24: Q. 3, 7, 9, 13, 17.
  - Ch. 24: Pr. 1, 3, 5, 7, 9, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 51, 53, 55, 57, 59, 67, 69, 71, 73, 75, 77, 81, 83, 85, 87.
- **Midterm Test: #2**
  - Second 60 min. test on Mon (Jul 14).
  - Will cover all material in Lectures 4–8 (except Ch. 25).
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut08**