UBC Physics 102

Lecture 6

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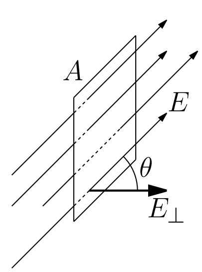
Outline

▷ Electric flux
▷ Gauss's law
▷ Applications
▷ End

Electric flux [Text: Sect. 22-1]

- **Discussion: Motivation**
  - In principle can always use Coulomb's law to find field.
  - But can get very difficult. Gauss's law often easier.
- **Definition:** *electric flux*



Electric flux, contd

- **Definition:** *electric flux, contd*
  - Flux is a measure of how much field "flows" through the surface.
  - Product of perpendicular component of electric field through a surface and the surface area,

$$\Phi_E = E_\perp A = E \cos\theta \, A.$$

  - Flux proportional to number of field lines that go through surface.
  - Magnitude of $E_\perp$ must be constant over surface.
- **Interactive Quiz: PRS 06a**

Electric flux, contd

- **Discussion: Closed surfaces**
  - Gauss's law deals with closed surfaces which enclose a volume of space.
  - For closed surfaces the net flux is the sum of the flux over each surface,

$$\Phi_E = \sum_{\text{surfaces}} E_\perp A.$$

  - Convention: $E_\perp$ is $+$ve coming out of volume, $-$ve going in.
- **Interactive Quiz: PRS 06b**

Electric flux, contd

- **Discussion: Field lines**
  - If every field line entering volume also leaves then flux is always zero (regardless of shape).
  - Flux only nonzero if field lines start or stop in volume.
  - Field lines only start/stop on charges.
  - So nonzero flux means volume encloses charge.

Gauss's law [Text: Sect. 22-2]

- **Definition:** *Gauss's law*
  - If a closed (Gaussian) surface contains a charge Q_{encl} then

$$\boxed{\sum_{\text{surfaces}} E_\perp A = \frac{Q_{\text{encl}}}{\epsilon_0}.}$$

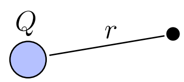
  - Q_{encl} is sum total of all charge inside surface.
  - Q_{encl} doesn't depend on positions or configuration of charges or on any charges outside surface (but electric field does).

Gauss's law, contd

- **Discussion: Gaussian surface**
  - Want to choose surface that makes problem easy to solve (usually trying to solve for E).
  - Need to know direction of $\mathbf{E}$ field.
  - Use symmetry.
  - Construct surface so field is parallel or perpendicular to it *everywhere*.
  - Parallel sections have zero flux, $\Phi_E = 0$.
  - Perpendicular sections have constant $E_\perp$ so $\Phi_E = E_\perp A$ is easy to compute.

Gauss's law, contd

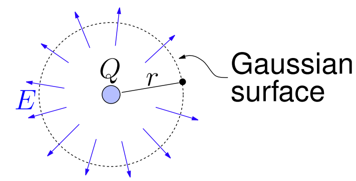
- **Derivation: Coulomb's law**
  - Can derive Coulomb's law from Gauss's law. Want to determine electric field at a distance r from a point charge Q.



  - System looks the same from any angle so electric field must also look the same from any angle (spherical symmetry).
  - E-field must point radially away from (or to) charge.
  - So pick Gaussian surface so that E-field is always perpendicular: a *sphere*.

Gauss's law, contd

- **Derivation: Coulomb's law, contd**

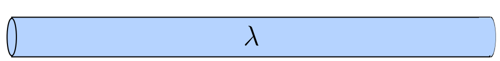


  - Enclosed charge: $Q_{\rm encl} = Q$.
  - Flux: $\Phi_E = E_\perp A = E(4\pi r^2)$ (surface area of sphere).
  - Gauss's law: $Q/\epsilon_0 = E(4\pi r^2)$ or

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}. \quad \square$$

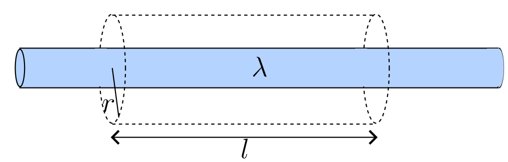
Applications [Text: Sect. 22-3]

- **Discussion: Gauss's law**
  - Gauss's law more general than Coulomb's.
  - Gauss's law one of Maxwell's equations: 4 laws that describe *all* electricity and magnetism.
- **Interactive Quiz: PRS 06c**
- **Example: Long, uniformly charged line**
  - A very long, straight wire possesses a uniform charge per unit length λ. Calculate the electric field at a distance r from the wire.



Applications, contd

- **Solution: Long, uniformly charged line**
  - By symmetry, the electric field must point radially out from the wire.
  - So a good choice for a Gaussian surface is a cylinder (length l, radius r).



  - Then $E \perp$ to side of cylinder and $\parallel$ to ends.
  - Enclosed charge: $Q_{\rm encl} = \lambda l$.

Applications, contd

- **Solution: Long, uniformly charged line, contd**
  - Flux: $(E_\perp A)_{\text{side}} = E\,2\pi r l$, $(E_\perp A)_{\text{ends}} = 0$.
  - Gauss's law: $\sum E_\perp A = Q_{\text{encl}}/\epsilon_0$,

$$\begin{aligned} E\,2\pi r l &= \frac{\lambda l}{\epsilon_0} \\ E &= \frac{\lambda}{2\pi\epsilon_0 r}. \end{aligned}$$

  - Could have solved with Coulomb's law but would have had to integrate.

End

- **Practice Problems:**
  - Ch. 22: Q. 1, 2, 3, 4, 5, 7, 9, 13.
  - Ch. 22: Pr. 3, 5, 7, 9, 17, 21, 23, 25, 27, 29, 31, 33, 39, 41, 43, 45, 47, 49.
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut06**