UBC Physics 102

Lecture 5 Version 2

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Outline

- ▷ Electric field
- ▷ Conductors
- ▷ Continuous charge distributions
- ▷ End

Electric field [Text: Sect. 21-6]

- **Definition:** *electric field*
  - If force $\mathbf{F}$ on test charge q then electric field $\mathbf{E}$ is

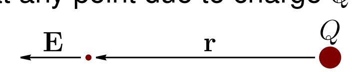
$$\mathbf{E} = \frac{\mathbf{F}}{q}.$$

  - Force depends on charge q but $\mathbf{E}$ is the same for *all test charges*.
  - So electric field is more useful quantity to work with. Once you know $\mathbf{E}$ can easily compute force on any test charge q via

$$\mathbf{F} = q\mathbf{E}.$$

Electric field, contd

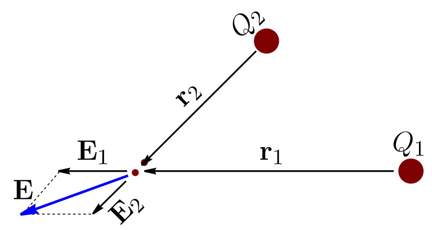
- **Definition:** *Coulomb's law*
  - Convenient to use electric field form of Coulomb's law.
  - Gives field at any point due to charge Q,



$$\mathbf{E} = k\frac{Q}{r^2}\hat{\mathbf{r}}.$$

Electric field, contd

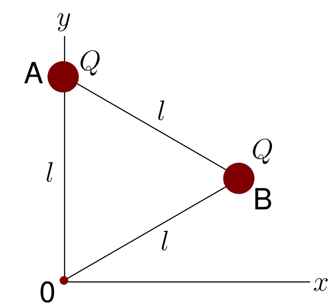
- **Discussion: Superposition principle**
  - If dealing with more than one charge, can just add up electric field due to each to calculate net electric field at a point,



$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots.$$

Electric field, contd

- **Example: Pr. 40**
  - Determine the electric field $\mathbf{E}$ at the origin 0 due to the two charges at A and B.



Electric field, contd

- **Solution: Pr. 40**
  - By the superposition principle

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_A + \mathbf{E}_B \\ &= E_A \hat{\mathbf{r}}_A + E_B \hat{\mathbf{r}}_B.\end{aligned}$$

  - From Coulomb's law the magnitudes of the electric fields are

$$E_A = E_B = \frac{kQ}{l^2}.$$

  - Just need to find the directions. Direction from A to origin is

$$\hat{\mathbf{r}}_A = -\hat{\mathbf{j}}.$$

Electric field, contd

- **Solution: Pr. 40, contd (Correction)**
  - B is at (x, y) where $y = \frac{1}{2}l$ and $x^2 + y^2 = l^2$ so $x = \sqrt{\frac{3}{4}}l$.
  - Direction from B to origin is

$$\hat{\mathbf{r}}_B = -\sqrt{\frac{3}{4}}\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}} = -\frac{\sqrt{3}}{2}\hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}}.$$

  - So net electric field at origin is

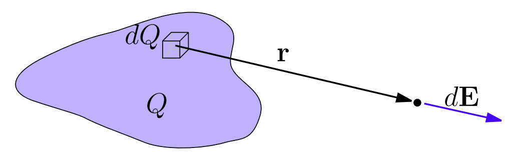
$$\begin{aligned}\mathbf{E} &= \frac{kQ}{l^2}(\hat{\mathbf{r}}_A + \hat{\mathbf{r}}_B) \\ &= \frac{kQ}{l^2}\left(-\frac{\sqrt{3}}{2}\hat{\mathbf{i}} - \frac{3}{2}\hat{\mathbf{j}}\right). \quad \square\end{aligned}$$

Conductors [Text: Sect. 21-9]

- **Interactive Quiz: PRS 05a**
- **Discussion: Conductors**
  - Conductors have free electrons.
  - Electrons move under force of electric field until the electric field is zero.
  - So electric field inside a conductor is always zero (after electrons have reached final position).
  - If a conductor has a net charge, it is always distributed on the surface, never in the interior the conductor.
- **Interactive Quiz: PRS 05b**

Continuous charge distributions [Text: Sect. 21-7]

- **Discussion: Continuous charges**
  - If object too large to be treated as point charge, can still solve for electric field.
  - Divide object into small chunks and add up field due to each chunk (superposition principle).



  - If small enough, each chunk obeys Coulomb's law

$$d\mathbf{E} = \frac{k\,dQ}{r^2}\hat{\mathbf{r}}.$$

Continuous charge distributions, contd

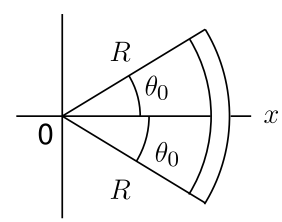
- **Discussion: Continuous charges, contd**
  - Total electric field is sum of all contributions

$$\mathbf{E} = \int d\mathbf{E}.$$

  - Method can be difficult but is guaranteed to work. Next class will show easier method that works in special cases.

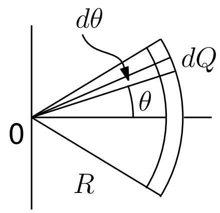
Continuous charge distributions, contd

- **Example: Pr. 49**
  - A thin rod bent into the shape of an arc of a circle of radius R carries a uniform charge per unit length λ. The arc subtends a total angle $2\theta_0$, symmetric about the x axis, as shown below. Determine the electric field $\mathbf{E}$ at the origin 0.



Continuous charge distributions, contd

- **Solution: Pr. 49**
  - We need to divide this continuous charge distribution into discrete chunks of charge dQ.
  - The obvious way is to take small increments of the angle $d\theta$.



  - Then the chunk has charge $dQ = \lambda R\, d\theta$.

Continuous charge distributions, contd

- **Solution: Pr. 49, contd**
  - Now we can apply Coulomb's law to get the electric field due to a single chunk,

$$d\mathbf{E} = \frac{k\,dQ}{R^2}(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}}).$$

  - We can sum over all the chunks to get the total electric field,

$$\begin{aligned}\mathbf{E} &= \int d\mathbf{E} \\ &= \frac{k}{R^2}\int dQ(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}})\end{aligned}$$

Continuous charge distributions, contd

- **Solution: Pr. 49, contd**

$$\begin{aligned}\mathbf{E} &= \frac{k}{R^2}\int_{-\theta_0}^{+\theta_0} \lambda R\, d\theta(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}}) \\ &= -\frac{k\lambda}{R}\left[\hat{\mathbf{i}}\int_{-\theta_0}^{+\theta_0}\cos\theta\, d\theta + \hat{\mathbf{j}}\int_{-\theta_0}^{+\theta_0}\sin\theta\, d\theta\right].\end{aligned}$$

  - Notice the $\hat{\mathbf{j}}$ contributions cancel out, leaving only

$$\begin{aligned}\mathbf{E} &= -\frac{k\lambda}{R}\hat{\mathbf{i}}\,2\sin\theta_0 \\ &= -\frac{2k\lambda\sin\theta_0}{R}\hat{\mathbf{i}}. \quad \square\end{aligned}$$

End

- **Practice Problems:**
  - Ch. 21: Q. 15, 17, 19, 21, 23
  - Ch. 21: Pr. 11, 13, 15, 19, 25, 27, 29, 35, 37, 39, 41, 43, 55, 57, 71, 73, 75, 77, 79, 81, 83, 87
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut05**