

(A2) in Otto and Whitton (2000) contains typos and should read:

$$\frac{\sigma_c \sigma_c'}{\ln \left[c N \left(e^{\frac{\sigma_c}{2 c N \nu \sigma_c'} - 1} \frac{\sigma_c'}{\sigma_c + \sigma_c} \right) \right]}$$

Note the c in the denominator and the σ_c' in the exponential. These errors were in the manuscript only and did not affect subsequent derivations for equations (3).

■ **Derivation of A2 of Otto and Whitton (2000)**

Letting $\sigma_1 = \sigma_c$ and $\sigma_2 = \sigma_c'$, the following derives equation A2, using the fixation probability given by equation A1:

$$\begin{aligned} & \text{Simplify}[p[t] \text{ c N } \nu \text{ fixation}[t] /. \text{fixation}[t] \rightarrow 2 \sigma_2 (\sigma_1 + \sigma_2) / (\sigma_2 + p[t] \sigma_1) /. \\ & \quad p[t] \rightarrow p_0 / (p_0 + (1 - p_0) \text{Exp}[-\sigma_1 t])] \\ & \frac{2 c e^{t \sigma_1} N p_0 \nu \sigma_2 (\sigma_1 + \sigma_2)}{\sigma_2 - p_0 \sigma_2 + e^{t \sigma_1} p_0 (\sigma_1 + \sigma_2)} \end{aligned}$$

The number of fixation events is given by the integral with respect to time:

$$\begin{aligned} & \text{Integrate}[\%, t] \\ & \frac{2 c N \nu \sigma_2 \text{Log}[\sigma_2 - p_0 \sigma_2 + e^{t \sigma_1} p_0 (\sigma_1 + \sigma_2)]}{\sigma_1} \end{aligned}$$

Evaluating the definite integral from time = 0 to g :

$$\begin{aligned} & (\% /. t \rightarrow g) - (\% /. t \rightarrow 0) \\ & - \frac{2 c N \nu \sigma_2 \text{Log}[\sigma_2 - p_0 \sigma_2 + p_0 (\sigma_1 + \sigma_2)]}{\sigma_1} + \frac{2 c N \nu \sigma_2 \text{Log}[\sigma_2 - p_0 \sigma_2 + e^{g \sigma_1} p_0 (\sigma_1 + \sigma_2)]}{\sigma_1} \end{aligned}$$

The “Log” here is the natural log.

Setting this to one (i.e., one fixation event) and solving for g gives the number of generations that must pass, on average, between the appearance of two successful beneficial mutations:

$$g /. \text{Simplify}[\text{Solve}[\% == 1, g]]$$

$$\left\{ \frac{\text{Log} \left[\frac{(-1+p_0) \sigma_2 + e^{2 c N \nu \sigma_2} (p_0 \sigma_1 + \sigma_2)}{p_0 (\sigma_1 + \sigma_2)} \right]}{\sigma_1} \right\}$$

The rate of fixation is the inverse of this quantity. Also setting the initial frequency to $1/(c*N)$ in an organism with ploidy level “ c ” gives:

$$\text{Simplify}[1 / \% /. p_0 \rightarrow 1 / (c * N)]$$

$$\left\{ \frac{\sigma_1}{\text{Log} \left[\frac{\sigma_2 - c N \sigma_2 + e^{2 c N \nu \sigma_2} (\sigma_1 + c N \sigma_2)}{\sigma_1 + \sigma_2} \right]} \right\}$$

Assuming that the population size is large (N large), we drop σ_2 relative to $c N \sigma_2$ in the above to get:

$$\frac{\sigma_1}{\text{Log} \left[c N \left(e^{\frac{\sigma_1}{2 c N \nu \sigma_2} - 1} \frac{\sigma_2}{\sigma_1 + \sigma_2} \right) \right]}$$

This gives the rate at which second beneficial alleles are expected to fix, which when multiplied by their beneficial effect σ_2 gives the rate of increase in fitness:

$$\mathbf{RateFitnessIncrease} = \frac{\sigma_1 \sigma_2}{\mathbf{Log} \left[c N \left(e^{\frac{\sigma_1}{2cNv\sigma_2}} - 1 \right) \frac{\sigma_2}{\sigma_1 + \sigma_2} \right]};$$

Thus (A2) contains two typos: The “c” is missing in the denominator, and it should be σ_2 in the denominator of the exponent term.

■ Confirming that equations (3) are unaffected

Assuming that the effects of the mutations are the same ($\sigma_1 = \sigma_2$), the effect of ploidy on rate of fitness increase is:

$$\mathbf{hap} = \mathbf{RateFitnessIncrease} /. c \rightarrow 1 /. \sigma_2 \rightarrow \sigma /. \sigma_1 \rightarrow \sigma$$

$$\frac{\sigma^2}{\mathbf{Log} \left[\frac{1}{2} \left(-1 + e^{\frac{1}{2Nv}} \right) N \right]}$$

$$\mathbf{dip} = \mathbf{RateFitnessIncrease} /. c \rightarrow 2 /. \sigma_2 \rightarrow h \sigma /. \sigma_1 \rightarrow h \sigma$$

$$\frac{h^2 \sigma^2}{\mathbf{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right]}$$

$$\mathbf{tetra} = \mathbf{RateFitnessIncrease} /. c \rightarrow 4 /. \sigma_2 \rightarrow h1 \sigma /. \sigma_1 \rightarrow h1 \sigma$$

$$\frac{h1^2 \sigma^2}{\mathbf{Log} \left[2 \left(-1 + e^{\frac{1}{8Nv}} \right) N \right]}$$

For (3a):

dip / hap

$$\frac{h^2 \mathbf{Log} \left[\frac{1}{2} \left(-1 + e^{\frac{1}{2Nv}} \right) N \right]}{\mathbf{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right]}$$

We take the limit in two cases.

When Nv is assumed very large, we obtain a limit of h^2 , using the following approximations (recall that $-1 + e^x \sim x$ for x small):

$$\mathbf{Log} \left[\frac{1}{2} \left(-1 + e^{\frac{1}{2Nv}} \right) N \right] \sim \mathbf{Log} \left[\frac{1}{2} \left(\frac{1}{2Nv} \right) N \right] = \mathbf{Log} \left[\left(\frac{1}{4Nv} \right) N \right]$$

$$\mathbf{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right] \sim \mathbf{Log} \left[\left(\frac{1}{4Nv} \right) N \right], \text{ which is the same as above.}$$

When Nv is assumed very small, we obtain a limit of $2h^2$, using the following approximations:

$$\mathbf{Log} \left[\frac{1}{2} \left(-1 + e^{\frac{1}{2Nv}} \right) N \right] \sim \mathbf{Log} \left[\frac{1}{2} \left(e^{\frac{1}{2Nv}} \right) N \right] = \mathbf{Log} \left[\frac{1}{2} \left(e^{\frac{1}{2Nv}} \right) N \right] = \mathbf{Log} \left[\frac{1}{2} N \right] + \frac{1}{2Nv} \sim \frac{1}{2Nv}$$

$$\mathbf{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right] \sim \mathbf{Log} \left[\left(e^{\frac{1}{4Nv}} \right) N \right] = \mathbf{Log} [N] + \frac{1}{4Nv} \sim \frac{1}{4Nv}$$

Similarly, for (3b):

tetra / dip

$$\frac{h1^2 \operatorname{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right]}{h^2 \operatorname{Log} \left[2 \left(-1 + e^{\frac{1}{8Nv}} \right) N \right]}$$

We take the limit in two cases.

When Nv is assumed very large, we obtain a limit of $\frac{h1^2}{h^2}$, using the following approximations (recall that $-1 + e^x \sim x$ for x small):

$$\begin{aligned} \operatorname{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right] &\sim \operatorname{Log} \left[\left(\frac{1}{4Nv} \right) N \right] \\ \operatorname{Log} \left[2 \left(-1 + e^{\frac{1}{8Nv}} \right) N \right] &\sim \operatorname{Log} \left[2 \left(\frac{1}{8Nv} \right) N \right], \text{ which is the same as above.} \end{aligned}$$

When Nv is assumed very small, we obtain a limit of $2 \frac{h1^2}{h^2}$, using the following approximations:

$$\begin{aligned} \operatorname{Log} \left[\left(-1 + e^{\frac{1}{4Nv}} \right) N \right] &\sim \operatorname{Log} \left[\left(e^{\frac{1}{4Nv}} \right) N \right] = \operatorname{Log} [N] + \frac{1}{4Nv} \sim \frac{1}{4Nv} \\ \operatorname{Log} \left[2 \left(-1 + e^{\frac{1}{8Nv}} \right) N \right] &\sim \operatorname{Log} \left[2 \left(e^{\frac{1}{8Nv}} \right) N \right] = \operatorname{Log} \left[2 \left(e^{\frac{1}{8Nv}} \right) N \right] = \operatorname{Log} [2N] + \frac{1}{8Nv} \sim \frac{1}{8Nv} \end{aligned}$$