

Appendix A from P.-O. Cheptou and F. Massol, ‘Pollination Fluctuations Drive Evolutionary Syndromes Linking Dispersal and Mating System’

(Am. Nat., vol. 174, no. 1, p. 46)

A Boundary Singular Strategy Is either Continuously Stable or neither Convergence nor Evolutionarily Stable

Equation (5) implies that, near a singular strategy (SS), the fitness of a rare mutant is a polynomial function of s' and d' of degree 2 or less, so that the second-order Taylor expansion formula is exact:

$$W_{s,d}(s', d') = W_{s,d}(s, d) + D^T \cdot \begin{pmatrix} s' - s \\ d' - d \end{pmatrix} + \frac{1}{2} \begin{pmatrix} s' - s \\ d' - d \end{pmatrix}^T \cdot \mathbf{H} \cdot \begin{pmatrix} s' - s \\ d' - d \end{pmatrix}, \quad (\text{A1})$$

where vector $D = (D_s, D_d)$ is the selection gradient and \mathbf{H} is the Hessian matrix given by

$$\mathbf{H} = \begin{pmatrix} \partial^2 W / \partial s'^2 & \partial^2 W / \partial s' \partial d' \\ \partial^2 W / \partial s' \partial d' & \partial^2 W / \partial d'^2 \end{pmatrix} (s, d). \quad (\text{A2})$$

However, the following relationships hold (by construction, see eq. [5]):

$$\frac{\partial^2 W_{s,d}}{\partial s'^2} = 0 = \frac{\partial^2 W_{s,d}}{\partial d'^2}, \quad (\text{A3})$$

$$\begin{aligned} \frac{\partial^2 W_{s,d}}{\partial s' \partial d'} = e & \frac{q(1/2 - \delta)(1 - e) - (1 - \delta)(1 - qe)}{(1 - \delta)(1 - d)s + dq[(1 - \delta)es + (1 - e)(1 - s\delta)]} \\ & + (1 - e) \frac{\delta eq - (1/2 - \delta)[1 - q(1 + e)]}{(1 - s\delta)(1 - d) + dq[(1 - \delta)es + (1 - e)(1 - s\delta)]}. \end{aligned} \quad (\text{A4})$$

In the particular cases where either $d = 0$ or $s = 0$, it can be shown that $\partial^2 W_{s,d} / \partial s' \partial d' < 0$ (the proof necessitates the separation of the cases $e < 2\delta - 1$ and $e > 2\delta - 1$ and that we use eqq. [7] and [9]). Thus, the Hessian matrix always has the following structure:

$$\mathbf{H} = \begin{pmatrix} 0 & -a \\ -a & 0 \end{pmatrix}, \quad (\text{A5})$$

where a is a positive function of the SS and of q , e , and δ .

We now remark that $W_{s,d}(s, d) = 1$ and that, at a boundary solution, the D vector consists of a negative component and a null component. We rename the two traits x and y , where x stands for the trait that is equal to 0 at the boundary SS (so $D_x(x, y) < 0$ if the boundary SS is convergence stable [CS]) and y stands for the other trait (so $D_y(x, y) = 0$). Let dx and dy be the differences between the mutant traits and the SS traits. Because $x = 0$, necessarily $dx > 0$. Equation (A1) can be rewritten

$$W_{x,y}(x + dx, y + dy) = 1 + (D_x - a \cdot dy) \cdot dx. \quad (\text{A6})$$

When dy is small, the fitness of the rare mutant is <1 when the boundary SS is CS, or it is >1 when the boundary SS is a repeller. This proves that any boundary SS is either a continuously stable strategy (CSS) or is neither CS nor an evolutionarily stable strategy.

Appendix B from P.-O. Cheptou and F. Massol, “Pollination Fluctuations Drive Evolutionary Syndromes Linking Dispersal and Mating System”

(Am. Nat., vol. 174, no. 1, p. 46)

Instability of the Interior Singular Strategy

Near an interior singular strategy (SS) (s, d) , selection gradient components are equal to 0, so that the fitness of a rare mutant deviating by Δs and Δd is in the form of equation (A1):

$$W_{s,d}(s + \Delta s, d + \Delta d) = 1 + b\Delta s\Delta d, \quad (\text{B1})$$

where b is a real negative number. Such a fitness function is the signature of a saddle point, and thus an interior SS is not an evolutionarily stable strategy (a mutant that has $\Delta s\Delta d < 0$ can invade the interior SS), nor is it convergence stable (projecting the selection gradient on the $\Delta s = -\Delta d$ line passing through the interior SS shows that there exists a direction on which the selection gradient points outward; see fig. 1B).

Appendix C from P.-O. Cheptou and F. Massol, ‘Pollination Fluctuations Drive Evolutionary Syndromes Linking Dispersal and Mating System’

(Am. Nat., vol. 174, no. 1, p. 46)

General Pollination Distribution

We explore the case of a continuous distribution of pollination proportions, that is, the proportion of outcrossing seeds fertilized is a stochastic variable, X , with mean $E(X) = \bar{X}$. We show in the following paragraphs that, whatever the assumed distribution of X , (1) interior singular strategies (SSs), if they exist, must be evolutionary repellers; and (2) boundary SSs can exist only on the d - and s -axes (i.e., for $s = 0$ or $d = 0$).

The fitness of the rare mutant can be computed as

$$\begin{aligned}
 w_{s,d}(s', d') &= E \frac{(1-d')\{(1-\delta)s' + [1 - (s+s')/2]X\} + qd'\{(1-\delta)s' + [1 - (s+s')/2]\bar{X}\}}{(1-d')[(1-\delta)s + (1-s)X] + qd[(1-\delta)s + (1-s)\bar{X}]} \\
 &= E \left(\frac{(1-d')\{(1-\delta)s' + [1 - (s+s')/2]X\} + qd'\{(1-\delta)s' + [1 - (s+s')/2]\bar{X}\}}{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]} \right. \\
 &\quad \times \left. \sum_{k=0}^{\infty} \left\{ \frac{-(1-s)(1-d)X}{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]} \right\}^k \right) \\
 &= \frac{(1-d')[1 - (s+s')/2]}{(1-s)(1-d)} + \sum_{k=0}^{\infty} \left\{ \frac{-(1-s)(1-d)}{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]} \right\}^k E(X^k) \quad (C1) \\
 &\quad \times \left(\frac{(1-d')(1-\delta)s' + qd'\{(1-\delta)s' + [1 - (s+s')/2]\bar{X}\}}{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]} - \frac{(1-d')[1 - (s+s')/2]}{(1-s)(1-d)} \right) \\
 &= 1 + \left(\frac{s'-s}{1-s} \right) \left(\frac{A(s,d)}{2C(s,d)} - \frac{1}{2} \right) + \left(\frac{d'-d}{1-d} \right) \left(\frac{A(s,d)}{B(s,d)} - 1 \right) + \frac{(s'-s)(d'-d)}{(1-s)(1-d)} \left(\frac{1}{2} - \frac{A(s,d)}{2F(s,d)} \right),
 \end{aligned}$$

where

$$A(s, d) = \sum_{k=0}^{\infty} E(X^k) \left\{ \frac{-(1-s)(1-d)X}{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]} \right\}^k, \quad (C2)$$

$$B(s, d) = \frac{1}{1 + (1-d)\{q[(1-\delta)s + (1-s)\bar{X}] - (1-\delta)s\}/\{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]\}}, \quad (C3)$$

$$C(s, d) = \frac{1}{1 + 2(1-s)\{(1-d)(1-\delta) + qd[1 - \delta - (\bar{X}/2)]\}/\{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]\}}, \quad (C4)$$

$$F(s, d) = \frac{1}{1 + 2\{(1-s)(1-d)[(1-\delta)(1-q) + q(\bar{X}/2)]\}/\{(1-d)(1-\delta)s + qd[(1-\delta)s + (1-s)\bar{X}]\}}. \quad (C5)$$

The expression $w_{s,d}(s', d')$ directly implies that no SS that is not situated on one of the four boundaries ($d = 0$,

$d = 1$, $s = 0$, $s = 1$) can be locally evolutionarily stable because its Hessian matrix has null diagonal elements; that is, near an interior SS,

$$w_{s,d}(s + \Delta s, d + \Delta d) = 1 + a\Delta s\Delta d, \quad (C6)$$

where $a = \{1/[(1-s)(1-d)]\}[(1/2) - (A(s,d))/(2F(s,d))]$ is a real number. Such a fitness function is the signature of a saddle point, and thus an interior SS cannot be evolutionarily stable. If an interior SS is convergence stable (CS), it is a branching point; otherwise, it is an evolutionary repeller. The selection gradient components are

$$D_d(s, d) = \frac{1}{1-d} \left(\frac{A(s, d)}{B(s, d)} - 1 \right), \quad (C7)$$

or, when $d = 1$,

$$D_d(s, 1) = 1 - \frac{(1-\delta)s}{q[(1-\delta)s + (1-s)\bar{X}]}, \quad (C8)$$

$$D_s(s, d) = \frac{1}{1-s} \left(\frac{A(s, d)}{2C(s, d)} - \frac{1}{2} \right), \quad (C9)$$

or, when $s = 1$,

$$D_s(1, d) = 1 - \frac{qd\bar{X}}{2(1-\delta)[1-d(1-q)]}. \quad (C10)$$

An interior SS has a null selection gradient; that is,

$$A(s, d) = B(s, d) = C(s, d). \quad (C11)$$

After some simplifications, the conditions $B(s, d) \leq C(s, d)$ and $B(s, d) \geq C(s, d)$ are equivalent to

$$g(d) = 1 - d(1-q) \leq \frac{q[(1-\delta)s + (1-s)\bar{X}]}{(1-\delta)(2-s)} = f(s),$$

$$g(d) = 1 - d(1-q) \geq \frac{q[(1-\delta)s + (1-s)\bar{X}]}{(1-\delta)(2-s)} = f(s), \quad (C12)$$

respectively. The study of function f allows the following statement: there is an interior SS if and only if $\bar{X} > 2(1-\delta)$. In particular, since $\bar{X} < 1$, an interior SS does not exist when $\delta < 0.5$. When an interior SS (d^* , s^*) exists, it is situated on the curve defined by $g(d^*) = f(s^*)$; that is,

$$d^* = \frac{1}{1-q} \left\{ 1 - \frac{q[(1-\delta)s^* + (1-s^*)\bar{X}]}{(1-\delta)(2-s^*)} \right\}. \quad (C13)$$

When $d > d^*$ and $s < s^*$, $g(d) < f(s)$ (since both functions are decreasing), and thus, $B(s, d) < C(s, d)$, so that the conditions $D_d(s, d) < 0$ and $D_s(s, d) > 0$ are impossible (these conditions would require $A(s, d) < B(s, d)$). Likewise, when $d < d^*$ and $s > s^*$, the conditions $D_d(s, d) > 0$ and $D_s(s, d) < 0$ are impossible. We have thus proved that any interior SS, if it exists, cannot be CS (since it does not attract points in its vicinity).

We now rapidly study the feasibility of boundary equilibriums. There can be four different boundary equilibriums:

1. $d = 0$, with $D_d < 0$ and $D_s = 0$, that is, $B(s, 0) > A(s, 0) = C(s, 0)$, thus implying $1 = g(0) > f(s)$;

2. $d = 1$, with $D_d > 0$ and $D_s = 0$, that is, $s < q\bar{X}/[(1 - \delta)(1 - q) + q\bar{X}]$ and $1 = A(s, 1) = C(s, 1)$, thus implying $(1 - s)[1 - \delta - (\bar{X}/2)] = 0$;

3. $s = 0$, with $D_s < 0$ and $D_d = 0$, that is, $c(0, d) > A(0, d) = B(0, d)$, thus implying $g(d) < f(0) = q\bar{X}/[2(1 - \delta)]$;

4. $s = 1$, with $D_s > 0$ and $D_d = 0$, that is, $d < [2(1 - \delta)]/[q\bar{X} + 2(1 - \delta)(1 - q)]$ and $1 = A(1, d) = B(1, d)$, thus implying $(1 - q)/[1 - d(1 - q)] = 0$ or $d = 0$.

These four conditions imply, respectively:

1. Either $\bar{X} < [2(1 - \delta)]/q$ or (exclusive) $s > [2(1 - \delta) - q\bar{X}]/[(1 - \delta)(1 - q) - q\bar{X}]$ (which is < 1);
2. $\bar{X} = 2(1 - \delta)$ and $s < 2q/(1 + q)$, which renders this boundary equilibrium unfeasible in all but one case;
3. Either $\bar{X} > [2(1 - \delta)]/q$ or (exclusive) $d > [1/(1 - q)](1 - \{q\bar{X}/[2(1 - \delta)]\})$ and $\bar{X} > 2(1 - \delta)$ (this equilibrium is unfeasible for $\bar{X} < 2(1 - \delta)$);
4. $q = 1$, which renders this boundary equilibrium unfeasible in all but one case.

From these, we conclude that only two ($d = 0$ or $s = 0$) of the four boundary equilibriums are robustly feasible.