Bio 301 Review Problems

(Additional review problems with answers are available on the course web page)

Probability Question:

Historic trapping data of arctic foxes suggests that the average density is 10 foxes/km^2. You sample a one km^2 grid and find only two foxes. You have a suspicion that the large shopping mall built in the neighborhood might have reduced the fox density.

(a) Calculate the probability that you would have observed so few foxes in the grid given the historic densities. To do this calculation, you must determine the probability of seeing two or fewer foxes.

(b) In the second year of your thesis, you return to the same site and resample the grid. Now you find 20 foxes. Based on the Poisson probability distribution, how many standard deviations is this above the expectation of 10?

(c) Offer two explanations for why you might have observed significantly fewer foxes in one year and significantly more in the following year.

Probability Question:

You are a doctor and have a patient that is at high risk of getting breast cancer (0.05 chance/year). She is considering getting a preventative mastectomy, which may reduce her risk of getting breast cancer by a factor of ten (0.005 chance/year). To help her decide, you give her the following information:

(a) The chance of getting breast cancer in the next ten years without a mastectomy is?

(b) The chance of getting breast cancer in the next ten years with a mastectomy is?

(c) Describe two probability distributions that could be used to answer this question and the assumptions underlying each.

Probability Question:

You are testing a new drug that is designed to prevent heart attacks in high-risk patients. You have studied a very large control group (no drugs) in which there have been 0.03 heart attacks/person/year. In the study, 100 people have been taking the drug for one year and none of them have had a heart attack. Given the rate of heart attacks among controls, how likely is it that nobody in your drug-treatment group would have a heart attack?

(a) Answer this question using a Binomial Distribution.

(b) Answer this question using a Poisson Distribution.

(c) Which answer, (a) or (b), should be more accurate and why?

Linear Model with Multiple Variables: Seed Germination

Light is a cue often used by small seeds to stimulate germination. The ratio of red to far red light is particularly important; far red light tends to inhibit germination and red light tends to stimulate it. This is important because when there are plants shading the seeds, these plants preferentially absorb red light, preventing germination. Once the shading plants are gone, however, red light passes and the seeds are stimulated to grow.

On day t, let $x(t)$ equal the number of seeds that are inhibited from germinating (dormant) and let $y(t)$ equal the number of seeds that are activated by red light and ready to germinate. Assume that a fraction, α, of dormant seeds become activated every day by exposure to red light and that a fraction, β, of activated seeds germinate each day (leaving the system). Also assume that activated seeds are not inactivated.

(a) Draw a flow diagram for this model.

(b) Write a transition matrix for the number of seeds of each type on day t+1 as a function of the number of seeds of each type on day t.

$$\begin{pmatrix} x(t+1) \\ \\ y(t+1) \end{pmatrix} = \begin{pmatrix} \qquad\qquad & \qquad\qquad \\ \\ & \end{pmatrix} \begin{pmatrix} x(t) \\ \\ y(t) \end{pmatrix}$$

(c) What are the eigenvalues for this matrix?

(d) What are the eigenvectors for this matrix?

(e) Write down the matrices, **D**, **A**, and $\mathbf{A}^{-1}$ that can be used to obtain a general solution for the model.

(f) Write down the exact general solution for the number of dormant and activated seeds on day t as a function of the initial numbers of seeds on day 0 ($x(0)$,$y(0)$). [Do not leave your answer as a product of matrices!]

(g) On the basis of part (f), what is the equilibrium number of dormant and activated seeds (make a reasonable assumption about the values of α and β).

(h) If germination of activated seeds is very rapid ($\beta >> \alpha$), then after a few days, what proportion of the seeds that remain would be dormant? [Hint: This is similar to the question of what fraction of the population would be at each age class in a demographic model.]

Linear Model with Multiple Variables: Stage-structured population

Consider an organism that has two size classes: small in the first year and large in the second year. Small individuals either die by the next census $(d_1 = 2/3)$ or survive to become large individuals ($p_1 = 1/3$). Large individuals never live for more than one year.

You have been censusing this poplation yearly and have found that the small individuals produce one offspring on average every year that survive until the next census ($m_0 = 1$). The large individuals are much more fertile and produce six offspring on average that survive until the next census ($m_1 = 6$).

(a) Write down the Leslie matrix for this population.

(b) What is the long-term rate of growth of this population?

(c) In the long-term, what proportion of the population is expected to be large?

(d) Would you expect it to take a long time or a short time for the proportion of large individuals to reach the value in (c)? Justify your answer.

Non-Linear Model with Multiple Variables: The Predator-Prey Model

In Lab 10, you simulated the Volterra predator-prey model. In continuous time, you observed cyclic dynamics where the cycle appeared to be neutrally stable (neither moving in nor out over time, with an orbit whose size depended on the initial conditions). In discrete time, however, you observed that the dynamics spiraled outwards, leading to the eventual extinction of the predators and/or prey. In both cases, prey increases were followed by predator increases, which were followed by a crash in the prey population.

We'll let $P(t)$ equal the number of predators at time t and $H(t)$ equal the number of prey. The parameters of the model are: the per capita growth of the prey in the absence of the predator (r), the per capita probability that a predator contacts and kills a prey (β), the per capita growth of the predator following the consumption of prey (c), and the death rate of predators (δ). With these definitions, the continuous-time predator-prey model is:

$$\mathrm{d}H/\mathrm{d}t = r\,H(t) - \beta\,H(t)\,P(t)$$

$$\mathrm{d}P/\mathrm{d}t = c\,H(t)\,P(t) - \delta\,P(t)$$

(a) Determine the two equilibria of these equations. [CAREFUL: Double check that the numbers of *both* predators and prey do not change over time when started at an equilibrium.]

(b) Determine the local stability matrix that approximates these equations near the equilibrium with both species absent. Repeat, finding the local stability matrix near the equilibrium with both species present.

(c) Find the eigenvalues for the two matrices in Part 2.

(d) From these eigenvalues, determine whether each equilibrium is stable or unstable, assuming that every parameter is positive. [NOTE: An equilibrium in a continuous-time model is stable if the real part of the eigenvalues are negative. The real part of a complex eigenvalue $a + b\,i$ is a.]

(e) Compare your answers from (a) – (d) here to Homework 9 based on the discrete-time model.