REVIEW QUESTIONS FOR ONE-VARIABLE MODELS

Question 1: During cell division, the very ends of our chromosomes (the telomeres) are not replicated, due to the fact that DNA replication starts from primers and the last primer on a chromosome is not replaced with DNA once it detaches from the chromosome. It is estimated that 50 basepairs are lost, on average, each cell generation from the chromosome.

(a) Write a recursion equation for chromosome length in a daughter cell (C[t+1]) as a function of the chromosome length in the parent cell (C[t]).

(b) Are there any equilibria to this equation?

[Note - During gamete production, a specific enzyme, telomerase, adds to the ends of the chromosomes, without which our chromosomes would eventually disappear.]

Question 2: The rate of transcription of many genes is regulated to ensure that sufficient levels of the gene products ($p[t]$) are maintained. One possible model of gene regulation assumes that the number of proteins produced per minute is inversely proportional to the level of the protein in the cell ($m/p[t]$), so that the rate of protein production is higher when protein levels are low.

(a) Assuming that a proportion, d, of the proteins degrade per minute, write down a recursion equation for the level of protein over time.

(b) What are the equilibria of this equation?

(c) Which of these equilibria is biologically valid?

(d) Determine what happens if the system is perturbed slightly away from the biologically valid equilibrium.

Question 3: Bulmer and Bull (1982) analyzed a multi-gene model for the evolution of the sex ratio (f = the proportion of females within a population). For a population with overlapping generations, the sex ratio changed over time according to the following differential equation:

$$df/dt = \alpha\ (1/2 - f[t])$$

where α is the "heritability" of sex ratio ($0 \leq \alpha \leq 1$).

(a) What is the equilibrium sex ratio of the above equation?

(b) What is the general solution for $f[t]$ given an initial sex ratio $f[0]$? [If you have the right answer and let t get large, the sex ratio will go to your answer from part (a).]

Question 4: Assume that the body produces insulin at a constant rate β per day that does not depend on the amount of insulin currently in the body. Once produced, these insulin molecules degrade at a constant rate α per day.

(a) Write down the differential equation that describes the change in number of insulin molecules (I) over time.

(b) What is the equilibrium of this equation?

(c) Is this equilibrium locally stable?

(d) What is the general solution, I[t]?

Question 5: In the selection models we've treated in class, we've assumed that the fitnesses are constant. In many real world examples, however, fitnesses depend on who else is in the population (= frequency dependent selection).

In a classic model in game theory, individuals are classified as "hawks" or "doves." Hawks always fight fiercely for resources whereas doves refuse to fight. Fighting reduces the fitness of hawks, but if a hawk and dove compete for food the hawk wins. Let V equal the value of the resource, which is shared between two individuals, and assume that this value is reduced by an amount C by fighting when two hawks encounter each other, where both parameters are positive. The following pay-off matrix describes the amount of resources gained by a hawk or a dove when competing with another individual for the resource:

	vs. Hawk	vs. Dove
Hawk	$(V-C)/2$	V
Dove	0	$V/2$

Assume that both hawks and doves have an equal base fitness of 1, which is altered slightly by this particular resource. Specifically, the overall fitness of a hawk is the base fitness plus the chance of encountering a hawk times the fitness gain of $(V-C)/2$ plus the chance of encountering a dove times the fitness gain of V. If the current frequency of hawks is p and the frequency of doves is $q = 1 - p$, the overall fitness of hawks is then $W_h = 1 + p\,(V - C)/2 + q\,V$. Similarly, the fitness of a dove is $W_d = 1 + q\,V/2$.

(a) Write down a recursion equation for the frequency of hawks in the next generation.

(b) Using the definition of a derivative, show that a differential equation describing the rate of change in the frequency of hawks is $\mathrm{d}p/\mathrm{d}t = p\,(1\text{-}p)\,(V - p\,C)/2$.

(c) What are the equilibria of this system?

(c) When is the interior equilibrium valid ($0 < p < 1$)?

(d) When is the interior equilibrium locally stable? When is it locally unstable?

(e) When the interior equilibrium is unstable, the population approaches a state with only hawks present ($p = 1$). In one sentence, explain why it makes sense that hawks would dominate the population in this case. [Use your results from (d).]