

1. Prove that the equilibrium allele frequency with heterozygote advantage (1-s:1:1-t) is equal to $t/(s+t)$.

The fastest way to prove this is to realize that with only selection acting on this locus, the equilibrium would be reached when the marginal fitnesses of the two alleles must be equal (otherwise selection would change the allele frequency and it is not an equilibrium).

The marginal fitness of the first allele is $w_A = 1 - ps$, and the marginal fitness of the second allele is $w_a = 1 - qt$. Our question then becomes "At what allele frequency is $1 - ps = 1 - qt$? Substitute $q = 1 - p$, to get

$$1 - ps = 1 - (1 - p)t$$

$$ps = (1 - p)t$$

$$ps + pt = t$$

$$p(s + t) = t$$

$$p = \frac{t}{s + t}$$

ALTERNATIVELY:

Use the formula $p' = \frac{p^2 w_{11} + pq w_{12}}{p^2 w_{11} + 2pq w_{12} + q^2 w_{22}}$, and find the equilibrium by setting p' to p . The relative fitness of each genotypes are $w_{11} = 1-s$, $w_{12} = 1$, and $w_{22} = 1-t$. So we start with

$$p' = \frac{p^2(1-s) + pq}{p^2(1-s) + 2pq + q^2(1-t)} = \frac{p(1-sp)}{1-sp^2-tq^2}$$

Let's solve for the change in allele frequency:

$$\begin{aligned}
p' - p &= \frac{p(1-sp)}{1-sp^2-tq^2} - p \\
&= \frac{p(1-sp)}{1-sp^2-tq^2} - p \frac{1-sp^2-tq^2}{1-sp^2-tq^2} \\
&= \frac{p(1-sp) - p(1-sp^2-tq^2)}{1-sp^2-tq^2} \\
&= \frac{p(-sp+sp^2+tq^2)}{1-sp^2-tq^2} \\
&= \frac{p(-spq+tq^2)}{1-sp^2-tq^2} \\
&= \frac{pq(-sp+tq)}{1-sp^2-tq^2}
\end{aligned}$$

We get an equilibrium whenever the change in allele frequency $p'-p$ is 0. From the last equation this will be true when $p = 0$, $q=0$, or $-sp+ tq=0$. The last condition is the one we are interested in. When is $-sp+ tq=0$?

$$-sp + tq = 0$$

$$-sp + t(1-p) = 0$$

$$-p(s+t) + t = 0$$

$$t = p(s+t)$$

$$p = \frac{t}{s+t}$$

which is the answer.

3. If the fitnesses of AA, Aa, and aa are 1.2, 0.9, and 0.6, and $p(0) = 0.7$, calculate $p(1)$, $p(2)$, and $p(3)$ for three generations of selection.

Use the formula $p' = \frac{p^2 w_{11} + pq w_{12}}{p^2 w_{11} + 2pq w_{12} + q^2 w_{22}}$. So after one generation of selection, the allele frequency will be

$$\begin{aligned}
p_{(1)} &= \frac{p_{(0)}^2 w_{11} + p_{(0)} q_{(0)} w_{12}}{p_{(0)}^2 w_{11} + 2 p_{(0)} q_{(0)} w_{12} + q_{(0)}^2 w_{22}} \\
&= \frac{0.7^2(1.2) + 0.7(0.3)(0.9)}{0.7^2(1.2) + 2(0.7)(0.3)(0.9) + 0.3^2(0.6)} \\
&= 0.7618
\end{aligned}$$

By the same equation (but remember that the mean fitness is changing in each generation) we get $p(2) = 0.81327$ and $p(3) = 0.85514$.