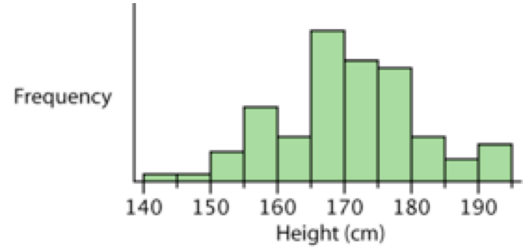
Describing data

Chapter 3

Two common descriptions of data

Location (or central tendency)

Width (or spread)



Measures of location

Mean
Median
Mode

Mean

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$$

n is the size of the sample

Mean

$Y_1 = 56$, $Y_2 = 72$, $Y_3 = 18$, $Y_4 = 42$

$\bar{Y} = (56+72+18+42) / 4 = 47$

Median

The *median* is the middle measurement in a set of ordered data.

The data:

18 28 24 25 36 14 34

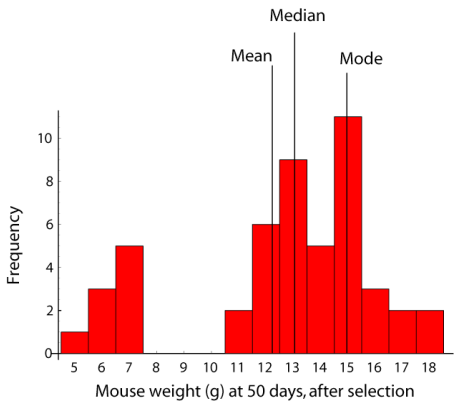
can be put in order:

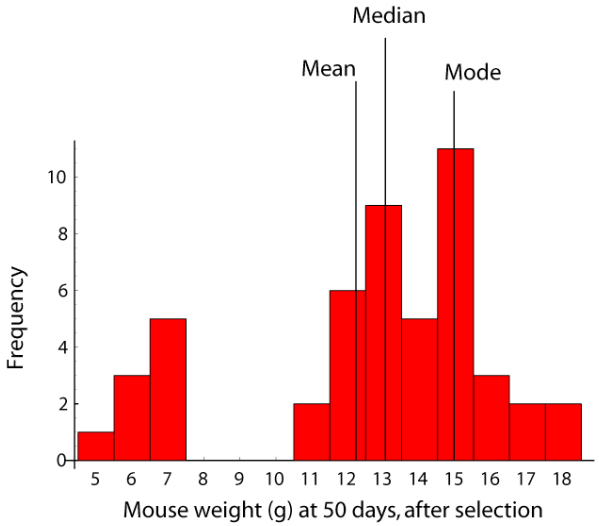
14 18 24 25 28 34 36

Median is 25.

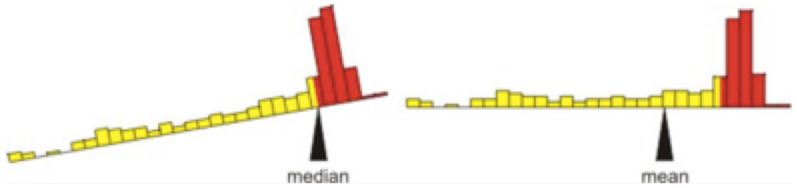
Mode

The mode is the most frequent measurement.





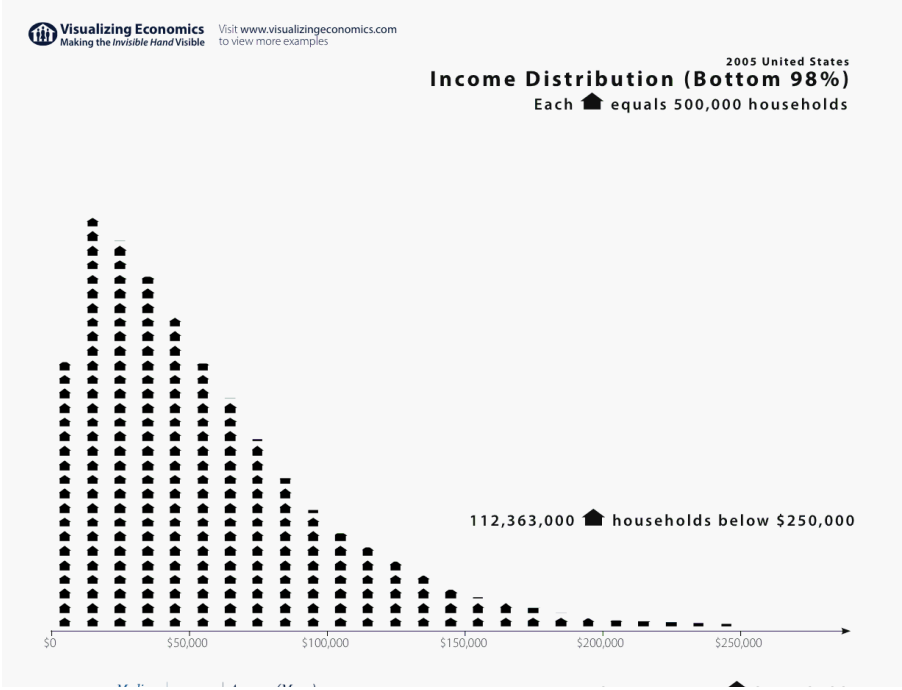
The mean is the center of gravity;
the median is the middle measurement.



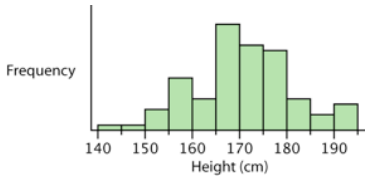
Mean and median for US household income, 2005

Median	$46,326
Mean	$63,344
Mode	$5000-$9999

Why?



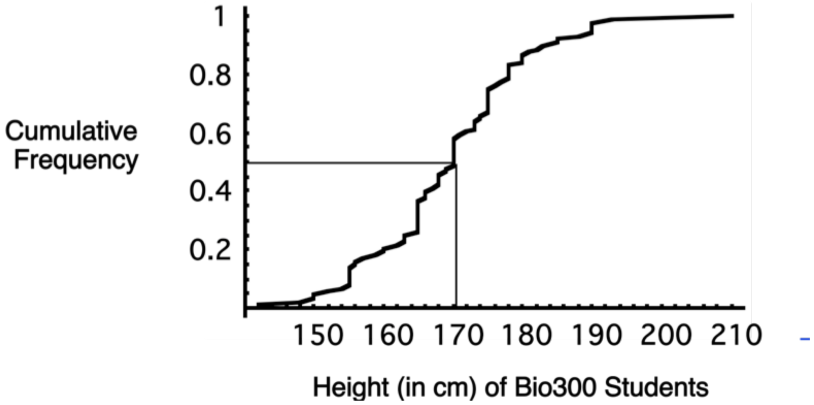
University student heights



Mean 169.3 cm

Median 170 cm

Mode 165-170 cm



Measures of width

- Range
- Standard deviation
- Variance
- Coefficient of variation

Range

14	17	18	20	22	22	24	
25	26	28	28	28	30	34	36

The range is the maximum minus the minimum:

36 -14 = 22

The range is a poor measure of distribution width

Small samples tend to give lower estimates of the range than large samples

So sample range is a ***biased estimator*** of the true range of the population.

Variance in a population

$$\sigma^2 = \frac{\sum_{i=1}^{N}(Y_i - \mu)^2}{N}$$

N is the number of individuals in the population.
μ is the true mean of the population.

Sample variance

$$s^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}$$

n is the sample size

Example: Sample variance

Family sizes of 5 BIOL 300 students: 2 3 3 4 4 (in units of siblings)

Y_i	$Y_i - \bar{Y}$	$(Y_i - \bar{Y})^2$
2	-1.2	1.44
3	-0.2	0.04
3	-0.2	0.04
4	0.8	0.64
4	0.8	0.64
Sums: 16		2.80

"Sum of squares"

$$\bar{Y} = \frac{2+3+3+4+4}{5} = \frac{16}{5} = 3.2$$

$$s^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}$$

$$s^2 = \frac{2.80}{4} = 0.70$$ (in units of siblings **squared**)

Shortcut for calculating sample variance

$$s^2 = \left(\frac{n}{n-1}\right)\left(\frac{\sum_{i=1}^{n} Y_i^2}{n} - \bar{Y}^2\right)$$

Example: Sample variance (shortcut)

Family sizes of 5 BIOL 300 students: 2 3 3 4 4

Y_i	Y_i^2	$Y_i - \bar{Y}$	$(Y_i - \bar{Y})^2$
2	4	-1.2	1.44
3	9	-0.2	0.04
3	9	-0.2	0.04
4	16	0.8	0.64
4	16	0.8	0.64
Sums: 16	54		2.80

$$\bar{Y} = \frac{2+3+3+4+4}{5} = 3.2$$

$$s^2 = \left(\frac{n}{n-1}\right)\left(\frac{\sum_{i=1}^{n} Y_i^2}{n} - \bar{Y}^2\right)$$

$$s^2 = \frac{5}{4}\left(\frac{54}{5} - (3.2)^2\right) = 0.70$$

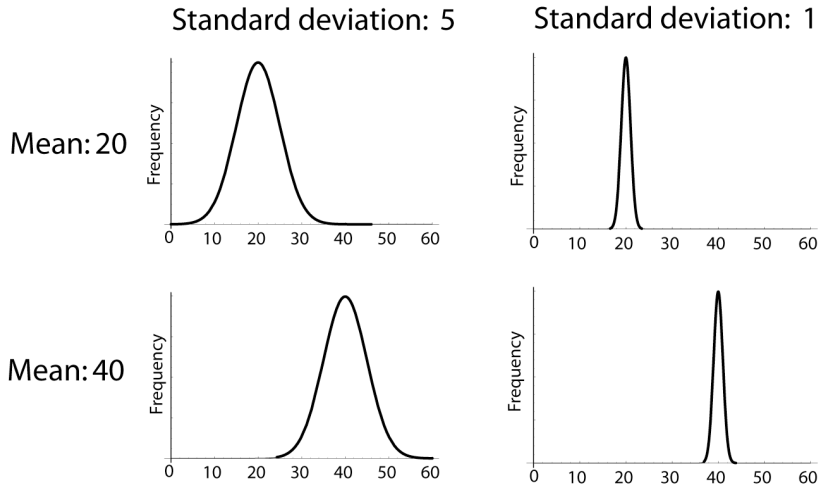
Standard deviation (SD)

Positive square root of the variance

σ is the true standard deviation
s is the sample standard deviation:

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}}$$

$s^2 = 0.70$ people^2
$s = \sqrt{0.70} = 0.84$ people



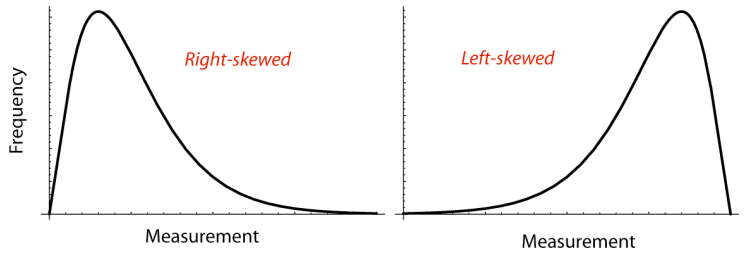
Coefficient of variation (CV)

$$CV = 100\% \frac{s}{\bar{Y}}$$

Skew

Skew is a measurement of asymmetry.

Skew (as in "skewer") refers to the pointy tail of a distribution



Nomenclature

	Population Parameters	Sample Statistics
Mean	μ	$\bar{Y}$
Variance	σ^2	s^2
Standard Deviation	σ	s

Basic stats in R

```
> mean(classHeightDataFull$height)
[1] 169.7955
> median(classHeightDataFull$height)
[1] 170
> sd(classHeightDataFull$height)
[1] 11.48828
> var(classHeightDataFull$height)
[1] 131.9807
```

Manipulating means

- The mean of the sum of two variables:

  $E[X + Y] = E[X] + E[Y]$

- The mean of the sum of a variable and a constant: $E[X + c] = E[X] + c$

- The mean of a product of a variable and a constant: $E[c X] = c E[X]$

Manipulating variance

- The variance of the sum of two variables:

  $Var[X + Y] = Var[X] + Var[Y]$

  if and only if X and Y are independent.

- The variance of the sum of a variable and a constant: $Var[X + c] = Var[X]$

- The variance of a product of a variable and a constant: $Var[c X] = c^2 Var[X]$

Example: converting units

Height:

Mean = 169.8 cm

Variance = 131.98 cm^2

In inches (1 cm = 0.394 in):

Mean: 169.8 cm × 0.394 = 66.9 in

Variance: 131.98 cm^2 $(0.394)^2$ = 20.5 in^2