ON THE NATURE OF THE STOCK MARKET: SIMULATIONS AND EXPERIMENTS

by

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Abstract

Over the last few years there has been a surge of activity within the physics community in the emerging field of *Econophysics*—the study of economic systems from a physicist's perspective. Physicists tend to take a different view than economists and other social scientists, being interested in such topics as phase transitions and fluctuations.

In this dissertation two simple models of stock exchange are developed and simulated numerically. The first is characterized by centralized trading with a market maker. Fluctuations are driven by a stochastic component in the agents' forecasts. As the scale of the fluctuations is varied a critical phase transition is discovered. Unfortunately, this model is unable to generate realistic market dynamics.

The second model discards the requirement of centralized trading. In this case the stochastic driving force is Gaussian-distributed "news events" which are public knowledge. Under variation of the control parameter the model exhibits two phase transitions: both a first- and a second-order (critical).

The decentralized model is able to capture many of the interesting properties observed in empirical markets such as fat tails in the distribution of returns, a brief memory in the return series, and long-range correlations in volatility. Significantly, these properties only emerge when the parameters are tuned such that the model spans the critical point. This suggests that real markets may operate at or near a critical point, but is unable to explain why this should be. This remains an interesting open question worth further investigation.

One of the main points of the thesis is that these empirical phenomena are not present in the stochastic driving force, but emerge endogenously from interactions between agents. Further, they emerge despite the simplicity of the modeled agents; suggesting complex market dynamics do not arise from the complexity of individual investors but simply from interactions between (even simple) investors.

Although the emphasis of this thesis is on the extent to which multi-agent models can produce complex dynamics, some attempt is also made to relate this work with empirical data. Firstly, the trading strategy applied by the agents in the

second model is demonstrated to be adequate, if not optimal, and to have some surprising consequences.

Secondly, the claim put forth by Sornette *et al.* [1] that large financial crashes may be heralded by accelerating precursory oscillations is also tested. It is shown that there is weak evidence for the existence of *log-periodic precursors* but the signal is probably too indistinct to allow for reliable predictions.

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