ON THE NATURE OF THE STOCK MARKET:
SIMULATIONS AND EXPERIMENTS

by

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Abstract

Over the last few years there has been a surge of activity within the physics community in the emerging field of Econophysics—the study of economic systems from a physicist’s perspective. Physicists tend to take a different view than economists and other social scientists, being interested in such topics as phase transitions and fluctuations.

In this dissertation two simple models of stock exchange are developed and simulated numerically. The first is characterized by centralized trading with a market maker. Fluctuations are driven by a stochastic component in the agents’ forecasts. As the scale of the fluctuations is varied a critical phase transition is discovered. Unfortunately, this model is unable to generate realistic market dynamics.

The second model discards the requirement of centralized trading. In this case the stochastic driving force is Gaussian-distributed “news events” which are public knowledge. Under variation of the control parameter the model exhibits two phase transitions: both a first- and a second-order (critical).

The decentralized model is able to capture many of the interesting properties observed in empirical markets such as fat tails in the distribution of returns, a brief memory in the return series, and long-range correlations in volatility. Significantly, these properties only emerge when the parameters are tuned such that the model spans the critical point. This suggests that real markets may operate at or near a critical point, but is unable to explain why this should be. This remains an interesting open question worth further investigation.

One of the main points of the thesis is that these empirical phenomena are not present in the stochastic driving force, but emerge endogenously from interactions between agents. Further, they emerge despite the simplicity of the modeled agents; suggesting complex market dynamics do not arise from the complexity of individual investors but simply from interactions between (even simple) investors.

Although the emphasis of this thesis is on the extent to which multi-agent models can produce complex dynamics, some attempt is also made to relate this work with empirical data. Firstly, the trading strategy applied by the agents in the
second model is demonstrated to be adequate, if not optimal, and to have some surprising consequences.

Secondly, the claim put forth by Sornette et al. [1] that large financial crashes may be heralded by accelerating precursory oscillations is also tested. It is shown that there is weak evidence for the existence of log-periodic precursors but the signal is probably too indistinct to allow for reliable predictions.
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4.18 In DSEM the price series does not get more regular as the system size is increased—in fact the fluctuation grow. This is especially true for $r_p = -0.75$ (a) but it is also indicated to a lesser degree at $r_p = 0.90$ (b).

5.1 Ten minute returns (86,000 data points) of the Swiss franc–U.S. dollar exchange rate [2] (negative tail) compared to power law with crossover to $\alpha \approx 3$ (a) and power law with exponential drop-off presented in this section (b).

5.2 Both tails of the cumulative distribution of daily (normalized) returns for the Nasdaq Composite index between October 1984 and Jun 2000 (4,000 data points) fit well to a decaying power law. The power law is truncated by two standard deviations in the positive tail but extends almost to four in the negative tail.

5.3 Scaling in the distribution of returns is only observed well below the critical point $\sigma_e \ll \sigma_c$ in CSEM as indicated by large values of the characteristic return $r_c$. For small $\sigma_e$ scaling occurs in both tails for daily returns but only for negative returns in monthly returns.

5.4 For $\sigma_e = 0.03$ in CSEM ($N = 1000$) the distribution of positive (monthly) returns (upper) almost converges to a Gaussian but still has a slightly heavy tail. The negative returns (lower), however, exhibit scaling for $r < r_c \approx 5.4$ with an exponent $\alpha \approx 1.1$.

5.5 DSEM only begins to exhibit scaling, as measured by a characteristic return exceeding three standard deviations, for price responses well below the first-order transition $r_2 = -0.33$ and as the price response approaches the critical point $r_1 = 1$. 
5.6 The characteristic returns in DSEM with a two-point distribution of price responses \((r_{lo} \text{ and } r_{hi})\) exceeds the required threshold of \(r_c = 3\) when \(r_{hi}\) is large (a). Neglecting the dependence on \(r_{lo}\) (b) it becomes clear that the characteristic return grows exponentially with the upper limit \(r_{hi}\), crossing the threshold near \(r_{hi} \approx 1\). .......................... 120

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5.9 Sample price series for DSEM Dataset 4 \((r_{lo} = 0.5, r_{hi} = 1.5)\) showing the price roughly tracks the exponential of the cumulative news \(e^\eta\). The proportionality constant is estimated from the data. ........................ 126

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5.13 A reproduction of Fig. 5.12 except with regularly sampled returns at an “hourly” interval (instead of tickwise). Short timescale anticorrelations crossing over to uncorrelated returns at long timescales are still observed so the effect is not an artifact of sampling tickwise. .......................... 130

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5.15 The Hurst exponent of the absolute returns, which measures the degree of clustered volatility, is strictly greater than one half for all parameter combinations in DSEM. It is particularly high when the upper limit of the two-point distribution \( r_{hi} \) is large or when the lower limit \( r_{lo} \) is small.

5.16 DSEM Dataset 4 \((N = 100\) agents\) is able to capture three important properties observed empirically when \( r_{lo} > 0.35 \) and \( r_{hi} > 1.25 \). The curves are contours from previous plots: (1) characteristic return \( r_c = 3 \) from Fig. 5.8 (solid line); (2) memory in return series = 100 from Fig. 5.14(a) (dashed line); and (3) Hurst exponent for the absolute returns \( H = 0.6 \) from Fig. 5.15 (dotted line).

5.17 Sample distribution of agents’ wealth from DSEM Dataset 3 \((N = 100, r_p = -0.50)\). There is insufficient data to distinguish between a normal and a log-normal distribution.

5.18 Sample distribution of agents’ wealth from DSEM Dataset 4 \((N = 100, r_{lo} = 0, r_{hi} = 1)\). The log-normal curves are calculated from each sub-population, revealing a strongly bimodal nature.

5.19 In DSEM with a two-point price response the wealth of each of the sub-populations \( w(r_p) \) depends strongly on the magnitude of the price response \(|r_p|\). The population with the smallest absolute price response \((r_{hi} \) to the left of zero and \( r_{lo} \) to the right) consistently has more wealth as indicated by the ratio of wealth between the two sub-populations.

6.1 Historical wealth using FIS versus (a) the Buy-and-Hold strategy and (b) the Nasdaq Composite Index over the same interval (rescaled to be equal at the start of the experiment).

6.2 Histograms of log-returns of capital \( r_{t+1} = \log(w_{t+1}/w_t) \) for both strategies. Notice BHS exhibits more large fluctuations (fatter tails) than FIS.

6.3 Sample fit of Eq. 6.29 to portfolio wealth on May 12, 2000. The best fit parameters indicate a crash is anticipated on or around \( t_c = July 4, 2000 \).

6.4 Daily wealth returns \((w_t/w_{t-1} - 1)\) are shown along with the dates forecasted to crash in (a). The qualities of the curve fits corresponding to the forecasted crashes, which suggest the reliability of the predictions, are shown in (b).

A.1 Comparison of weightings using standard and discounted windows.
A.2 Discounted least-squares fitting has a computational storage advantage over moving windows of \( N \) data points when \( N > M^2 + M + 2 \) where \( M \) is the number of parameters to be fitted.  

B.1 When a random walk is generated at some regular interval and sampled at another, \( \Delta \), the number of jumps between samples will vary.  

B.2 The kurtosis is only zero at integer values of the sampling interval \( \Delta \) and diverges as the sampling interval approaches zero.  

B.3 The distribution of increments for the random walk appears to have fatter tails than a normal distribution with the same variance when sampled at intervals of \( \Delta = 1.05 \). However, the tails still drop off as \( e^{-\frac{x^2}{2}} \).  

B.4 Discrete Brownian motion with Poisson-distributed jump intervals has tails which fall off exponentially (with a decay constant of 0.72), instead of as \( e^{-\frac{x^2}{2}} \), when sampled at regular intervals (\( \Delta = 1 \)).  

C.1 Sample fractional Brownian motion time series with different Hurst exponents: antipersistent \( H = 0.1 \) (top) has negative long-range correlations, uncorrelated \( H = 0.5 \) (center) is standard Brownian motion, and persistent \( H = 0.9 \) (bottom) has positive long-range correlations.  

C.2 Power spectral densities for the fractional Brownian motion time series shown in Fig. C.1. The points are from finite samples of 1000 points each and the line represents the theoretical spectrum. For low frequencies the power spectrum is well approximated by a power law \( 1/f^{2H+1} \).  

C.3 Scaled window variance analyses for the fractional Brownian motion time series shown in Fig. C.1 (exact \( H = 0.1, 0.5, \) and 0.9, respectively). The estimated values of \( H \) shown represent the best fit slopes of the lines. The analysis used \( M_{\text{min}} = 4 \) (see the text).  

C.4 Comparison of Hurst estimators using synthetic datasets of 1000 points each. The scaled-window variance method (SWV, *) performs significantly better than rescaled range analysis (\( R/S, + \)) and marginally better than dispersional analysis (Disp., \( \times \)). (The points are offset slightly to improve readability.)
C.5 Comparison of Hurst estimators on uncorrelated Lévy flight with characteristic exponent $\alpha$ using synthetic datasets of 1000 points each. Rescaled range ($R/S$, $\pm$) and dispersional analysis (Disp., $\times$) perform well but scaled window variance analysis (SWV,$\ast$) performs poorly, especially for small $\alpha$, tending towards the $1/\alpha$ curve. (The points are offset slightly to improve readability.)

C.6 Schematic representation of relation between fractional Brownian motion and Lévy flight. Traditional Brownian motion sits at the intersection ($H = 1/2$, $\alpha = 2$). The natural extension into the two-space is fractional Lévy motion which has correlated, non-Gaussian increments.
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