Physics 153 Section T0H - Week 8

RC Circuits

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1 Comments

Some of you really bombed the last assignment so I'll let you hand it in again (with some late penalties).

2 RC Circuits

In RC circuits there is a lag as the charge accumulates/dissipates on the capacitor. So, instead of constant voltages, currents, etc., all these properties approach their final values exponentially:

$$
\begin{aligned}
Q(t) - Q(\infty) &= [Q(0) - Q(\infty)]e^{-t/\tau} && (1) \\
I(t) - I(\infty) &= [I(0) - I(\infty)]e^{-t/\tau} && (2) \\
V(t) - V(\infty) &= [V(0) - V(\infty)]e^{-t/\tau}. && (3)
\end{aligned}
$$

Notice you only need to memorize one form of equation which applies to all these properties.

The time constant is

$$\tau = RC \qquad (4)$$

where C is the capacitance and R is the resistor(s) which *are in series with the capacitor* (I think).

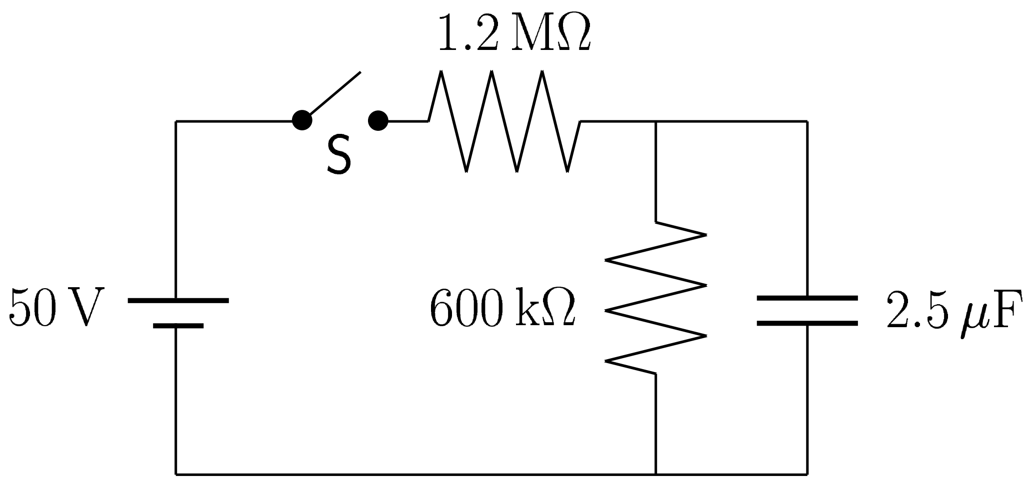
2.1 Initial and final values

The trick to solving the exponential equations is figuring out what each of the initial and final values are. Here are some hints:

1. The current across a capacitor always goes down to zero $I_C(\infty) = 0$.
2. Without a voltage source driving them, capacitors discharge $Q(\infty) = 0$.
3. If the capacitor has no charge on it, it acts like a short circuit (wire with no resistance).

3 Example

(From Tipler Ch. 23 #61.)



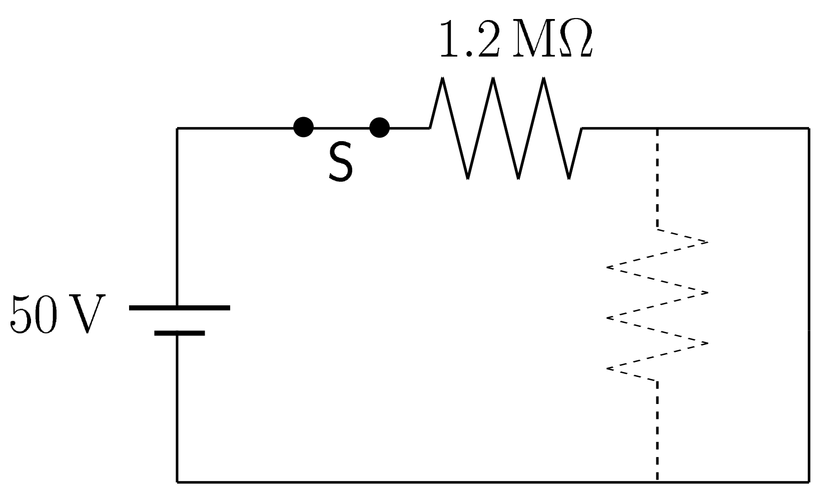
For the circuit above, (a) what is the initial battery current immediately after switch S is closed? (b) What is the battery current a long time after switch S is closed? (c) If the switch has been closed for a long time and is then opened, find the current through the $600\,\mathrm{k}\Omega$ resistor as a function of time.

4 Solution

4.1 Part (a)

"What is the initial battery current immediately after switch S is closed?"

Initially there is no charge on the capacitor so it is a short circuit and no current goes through the $600\,\mathrm{k\Omega}$ resistor:



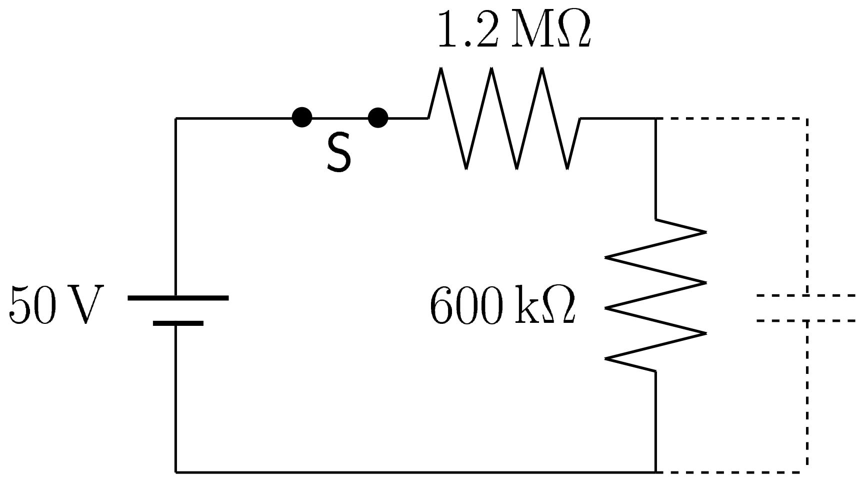
So the current through the battery is just

$$I_V(0) = \frac{V}{R} = \frac{50}{1.2 \times 10^6} = 41.7\,\mu\text{A}. \tag{5}$$

4.2 Part (b)

"What is the battery current a long time after switch S is closed?"

After a long time the capacitor current goes to zero so we basically just have a battery and two resistors in series:



So now the current across the battery is just

$$I_V(\infty) = \frac{V}{R} = \frac{50}{1.8 \times 10^6} = 27.8\,\mu\text{A}. \tag{6}$$

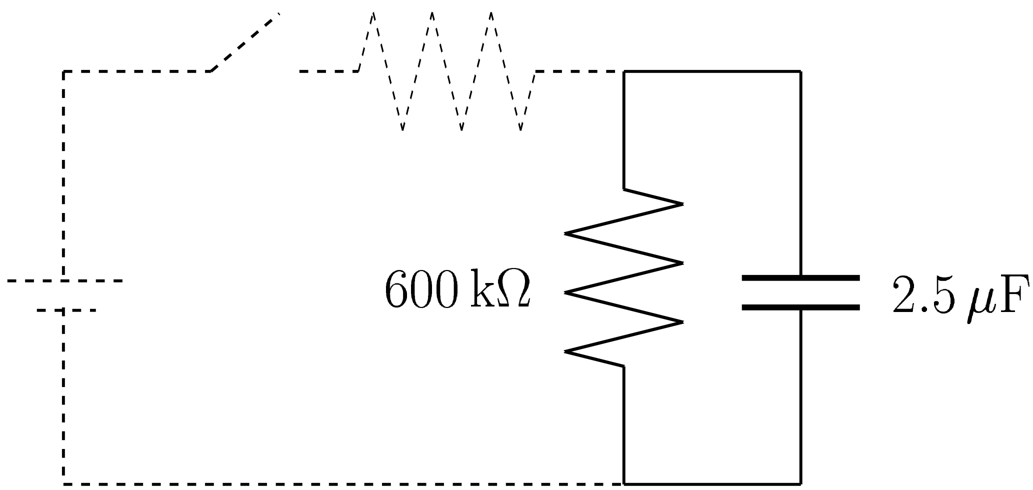
4.3 Part (c)

"If the switch has been closed for a long time and is then opened, find the current through the $600\,\Omega$ resistor as a function of time."

After a long time the voltage across the battery (or the $600\,\Omega$ resistor) has built up to

$$V_C(\infty) = I_V(\infty)R = 27.8 \cdot 600 = 16.7\,\mathrm{V}. \qquad (7)$$

Now we reset our clock to zero and open the switch. Then we have the following circuit with just a resistor and a capacitor:



The capacitor starts with the final voltage it had when the switch was closed

$$V_C'(0) = V_C(\infty) = 16.7\,\mathrm{V} \qquad (8)$$

so the initial current through the resistor is

$$I'(0) = \frac{V_C'(0)}{R} = \frac{16.7}{600} = 27.8\,\mu\text{A}. \tag{9}$$

(The *prime* denotes stuff after the switch was opened.)

The capacitor (and resistor) current must eventually go to zero so $I'(\infty) = 0$ and the time constant of the circuit is just

$$\tau = RC = 600 \cdot 2.5 = 1.5\,\text{s}. \tag{10}$$

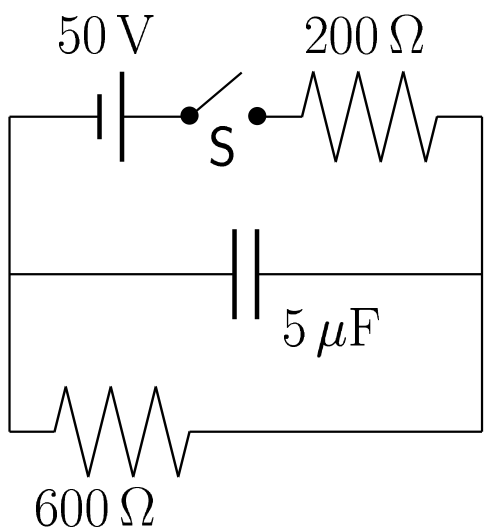
So we have all the bits we need to fill in the equation:

$$I'(t) - I'(\infty) = [I'(0) - I'(\infty)]e^{-t/\tau} \tag{11}$$

$$I'(t) = (27.8\,\mu\text{A})e^{-t/(1.5\,\text{s})}. \tag{12}$$

5 Assigned Problem

(From Tipler Ch. 23 #60.)



For the above circuit, (a) what is the initial battery current immediately after switch S is closed? (b) What is the battery current a long time after switch S is closed? (c) What is the current in the $600\,\Omega$ resistor as a function of time?