Physics 153 Section T0H - Week 11
LR Circuits

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1 Revenge!

Bring your pencils next week so you can fill out my teaching evaluation. Finally, your chance to *grade* me! Don't worry, I won't see them until well after the term is over.

2 Self-inductance

When a circuit has an inductor in it, the inductor responds to changes in the current by setting up an emf

$$V_L = -L\frac{dI}{dt} \tag{1}$$

which opposes the change in current. L is the self-inductance, measured in Henries (H).

3 LR Circuits

The opposition to change causes the current to change exponentially over time instead of instantly,

$$I(t) - I(\infty) = \left[I(0) - I(\infty)\right] e^{-t/\tau}. \tag{2}$$

(This equation applies to any part of the circuit, not just through the inductor.)

This should look familiar. It's the exact same equation I gave you for RC circuits. The only difference is in calculating the constants.

The time constant is given by

$$\tau = L/R. \tag{3}$$

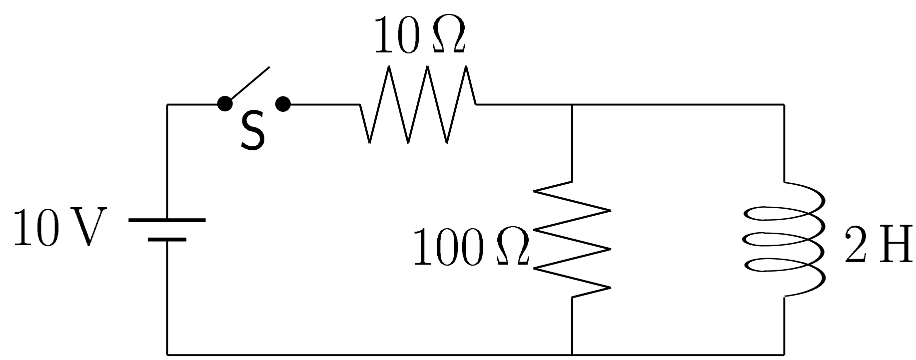
To calculate the initial and final values keep these facts in mind:

1. Initially, after a change in the circuit, the current across the inductor does not change. It stays the same as just before the change.

2. After a long time, the inductor just acts like a short-circuit.

4 Example

(From Tipler Ch. 26 #57.)



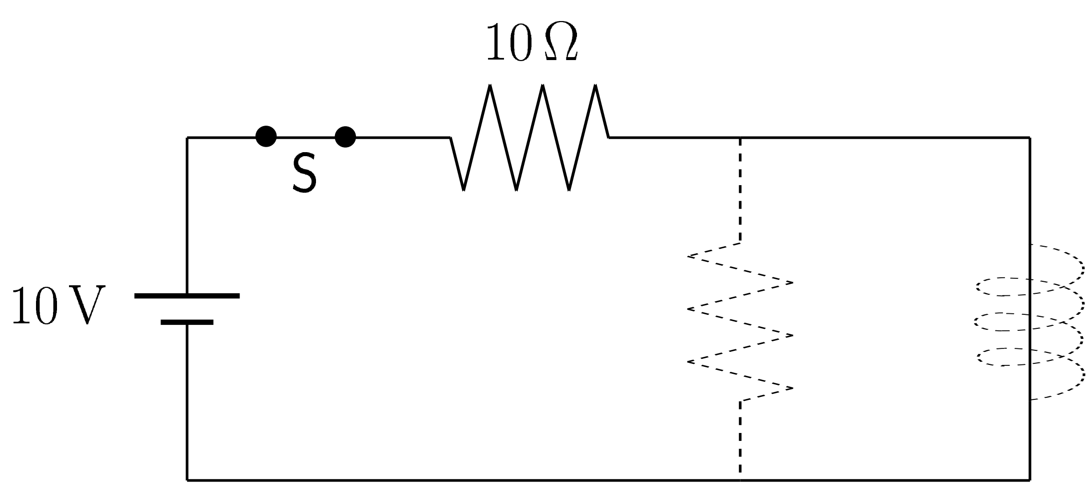
Given the circuit shown above, assume that switch S has been closed for a long time so that steady currents exist in the circuit and that the resistance of the inductor L may be considered to be zero. (a) Find the battery current, the current in the $100\,\Omega$ resistor, and the current through the inductor. (b) Find the initial voltage across the inductor when switch S is opened. (c) Give the current in the inductor as a function of time measured from the instant of opening switch S.

5 Solution

5.1 Part (a)

"Find the battery current, the current in the $100\,\Omega$ resistor, and the current through the inductor."

We're considering the circuit a long time after the switch has been closed (let's call this time $t = \infty$) so the inductor just acts as a short-circuit across the resistor so the circuit can be drawn as:



So the circuit just consists of a battery in series with a single resistor and the currents through the battery and the inductor are

$$I_V(\infty) = I_L(\infty) = \frac{V}{R_1} = \frac{10}{10} = 1\,\mathrm{A}. \tag{4}$$

Since the $100\,\Omega$ resistor is short-circuited, no current passes through it,

$$I_R(\infty) = 0. \tag{5}$$

5.2 Part (c)

"Give the current in the inductor as a function of time measured from the instant of opening switch S."

Note that I'm solving part (c) *before* part (b). That's because I can use the result from part (c) to solve part (b).

When we open the switch let's reset the clock to zero ($t = 0$) and denote everything after that with primes.

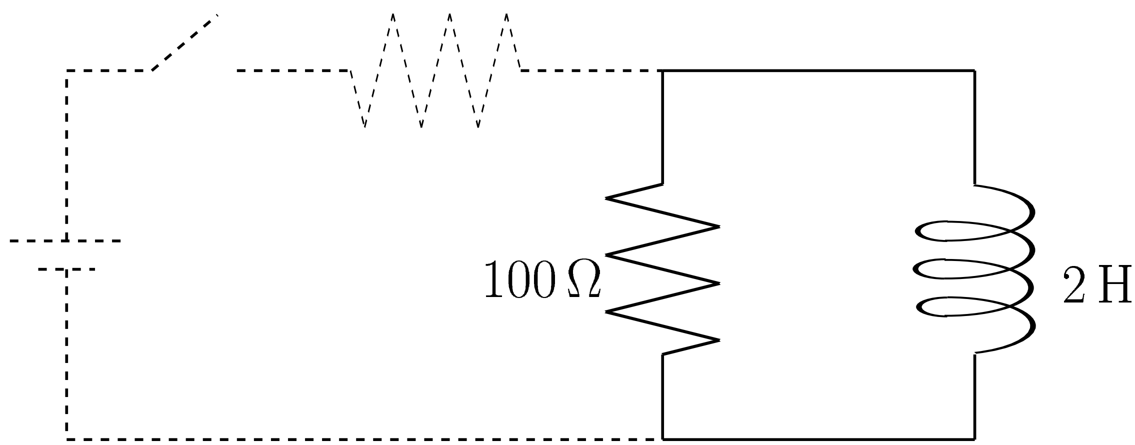
Recall, the general equation for the current through the inductor is

$$I'(t) - I'(\infty) = [I'(0) - I'(\infty)]\, e^{-t/\tau} \tag{6}$$

so we just need to fill in the constants.

5.2.1 Time constant

First, let's calculate the time constant. The circuit now looks like this:



So, the time constant just depends on the inductor and the $100\,\Omega$ resistor:

$$\tau = \frac{L}{R} = \frac{2}{100} = 0.02\,\mathrm{s}. \quad (7)$$

5.2.2 Initial current

Initially, the inductor resists a change in the current so it is the same as it was just before the switch was opened,

$$I'_L(0) = I_L(\infty) = 1\,\mathrm{A}. \quad (8)$$

5.2.3 Final current

After a long time the current must go to zero because there is no voltage driving it,

$$I'_L(\infty) = 0. \tag{9}$$

5.2.4 Current equation

Substituting all that into Eq. (6) gives

$$\begin{aligned} I'_L(t) - 0 &= [(1\,\mathrm{A}) - 0]\, e^{-t/(0.02\,\mathrm{s})} && (10) \\ I'_L(t) &= (1\,\mathrm{A}) e^{-t/(0.02\,\mathrm{s})}. && (11) \end{aligned}$$

5.3 Part (b)

"Find the initial voltage across the inductor when switch S is opened."

The voltage across the inductor is always given by

$$V_L = -L\frac{dI_L}{dt} \tag{12}$$

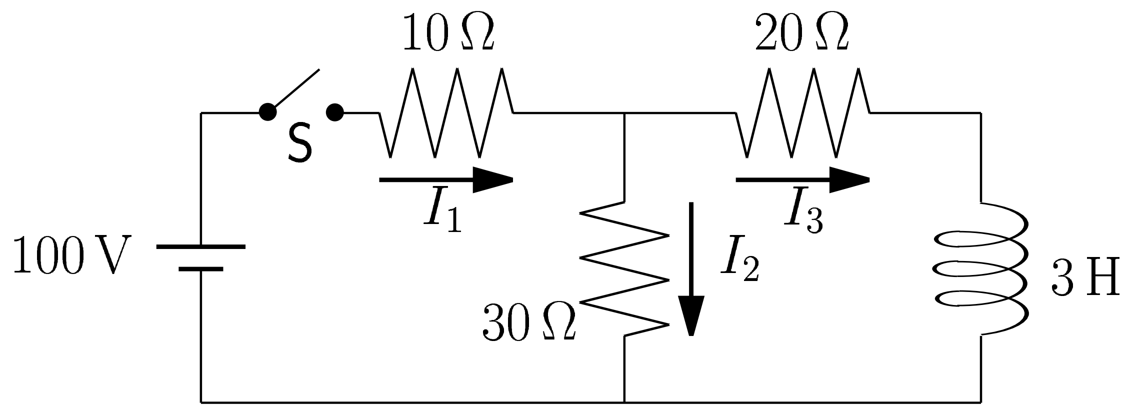
so we can just take the derivative of the current equation to get the voltage:

$$\begin{aligned} V_L'(t) &= -L\frac{d}{dt}\left[I_L'(0)e^{-t/\tau}\right] && (13) \\ &= -LI_L'(0)e^{-t/\tau}\frac{-1}{\tau} && (14) \\ &= \frac{LI_L'(0)}{\tau}e^{-t/\tau}. && (15) \end{aligned}$$

So, at time zero,

$$V_L'(0) = \frac{LI_L'(0)}{\tau} = \frac{(2)(1)}{0.02} = 100\,\mathrm{V}. \tag{16}$$

6 Assigned Problem



For the circuit shown above, find the currents I_1, I_2, and I_3 (a) immediately after switch S is closed and (b) a long time after switch S has been closed. After the switch has been closed for a long time, it is opened. Find the three currents (c) immediately after switch S is opened and (d) a long time after switch S was opened.