Physics 153 Section T0H - Week 10

Faraday's Law

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1 Comments

I've put your homework marks up on the web page (`www.physics.ubc.ca/~blok/phys153`). Remember, you're only allowed to skip 2 assignments. (I do accept late assignments—with penalties.)

2 Faraday's law

As the magnetic flux going through a conducting loop changes, it induces an *electromotive force* (emf) or voltage in the loop. The strength of the emf is given by Faraday's law:

$$\varepsilon = -\frac{d\phi_m}{dt} \tag{1}$$

The magnetic flux is given by

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}. \tag{2}$$

But Faraday's law is only really used in simple cases when the flux can be reduced to

$$\phi_m = BA\cos\theta \tag{3}$$

where θ is the angle between $\mathbf{B}$ and $\mathbf{A}$.

This stuff looks a lot like Gauss's law or Ampère's law but remember, in this case the loop is real, it can't be chosen arbitrarily.

3 Lenz's law

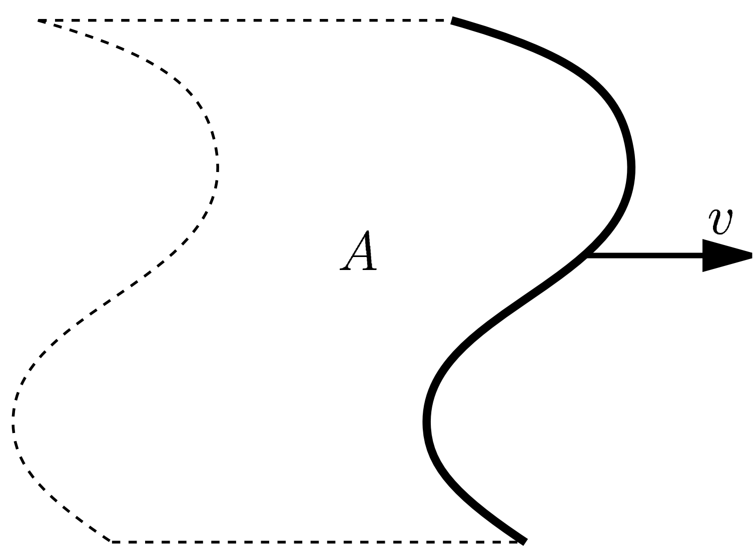
Lenz's law tells us which direction the induced current goes. Basically, the loop just wants to keep the magnetic flux constant.

So, if the flux is increasing, then the loop generates a current which generates its own magnetic field which counteracts the increasing flux and you can use the right-hand rule to figure out which direction the current goes.

4 Motional Emf

When we just have a conducting wire segment instead of a complete loop you use the same technique except it only applies when the wire is moving.

Then A is the area the wire has covered.



5 Aside: Maxwell's equations

Maxwell unified electromagnetism with the four fundamental equations below. In doing so he explained what light was.

"And Maxwell said...

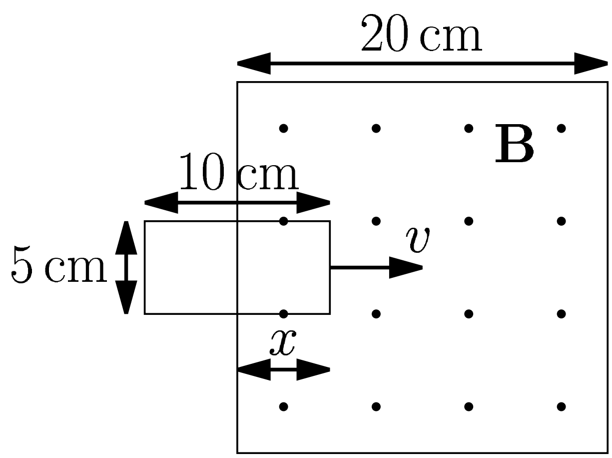
$\oint \mathbf{E} \cdot d\mathbf{A} = Q_{enc}/\epsilon_0$ (Gauss's law)	$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_m}{dt}$ (Faraday's law)
$\oint \mathbf{B} \cdot d\mathbf{A} = 0$	$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$ (Ampère's law)

...and there was light and the light was good."

(Just kidding! Don't sue me!)

6 Example

(From Tipler Ch. 26 #65.)



A $10\,\mathrm{cm}$ by $5\,\mathrm{cm}$ rectangular loop with resistance $2.5\,\Omega$ is pulled through a region of uniform magnetic field $B = 1.7\,\mathrm{T}$ (above) with constant speed $v = 2.4\,\mathrm{cm/s}$. The front of the loop enters the region at time $t = 0$. (a) Find and graph the flux through the loop as a function of time. (b) Find and graph the induced emf and the current in the loop as functions of time. Neglect any self inductance of the loop and extend your graphs from $t = 0$ to $t = 16\,\mathrm{s}$.

7 Solution

7.1 Part (a)

"Find and graph the flux through the loop as a function of time."

Notice that we can split the problem into 4 parts: (1) the loop is entering the magnetic field, (2) the whole loop is inside the magnetic field, (3) the loop is leaving the magnetic field, and (4) the loop is outside the magnetic field. The transitions occur when the ends of the loop reach the boundary of the magnetic field and are given by the times:

$$t_1 = (10\,\mathrm{cm})/v = 4.17\,\mathrm{s} \tag{4}$$
$$t_2 = (20\,\mathrm{cm})/v = 8.33\,\mathrm{s} \tag{5}$$
$$t_3 = (30\,\mathrm{cm})/v = 12.5\,\mathrm{s}. \tag{6}$$

For $t < t_1$ the loop is entering the magnetic area so

$$\phi_m(t < t_1) = BA = Bwvt \tag{7}$$

where $w = 5\,\mathrm{cm}$ is the width of the loop.

For $t_1 < t < t_2$ the loop has a constant magnetic flux

$$\phi_m(t_1 < t < t_2) = BA = Bwl \tag{8}$$

where $l = 10\,\mathrm{cm}$ is the length of the loop.

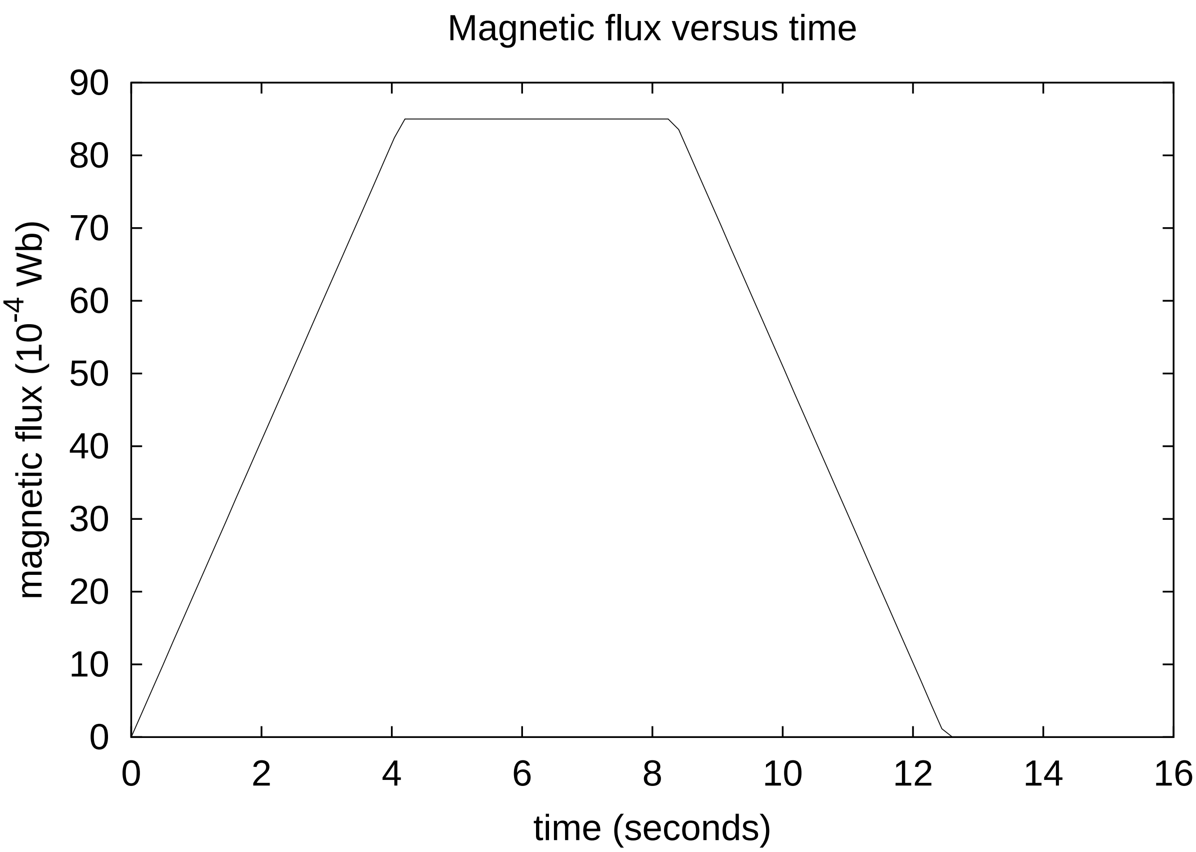
For $t_2 < t < t_3$ the loop is leaving the magnetic area so

$$\phi_m(t_2 < t < t_3) = BA = Bwl - Bwv(t - t_2) \tag{9}$$
$$= Bw(l + vt_2 - vt). \tag{10}$$

Finally, for $t > t_3$ the loop is totally outside the magnetic area so

$$\phi_m(t > t_3) = 0. \tag{11}$$

So the graph looks like...



7.2 Part (b)

"Find and graph the induced emf and the current in the loop as functions of time."

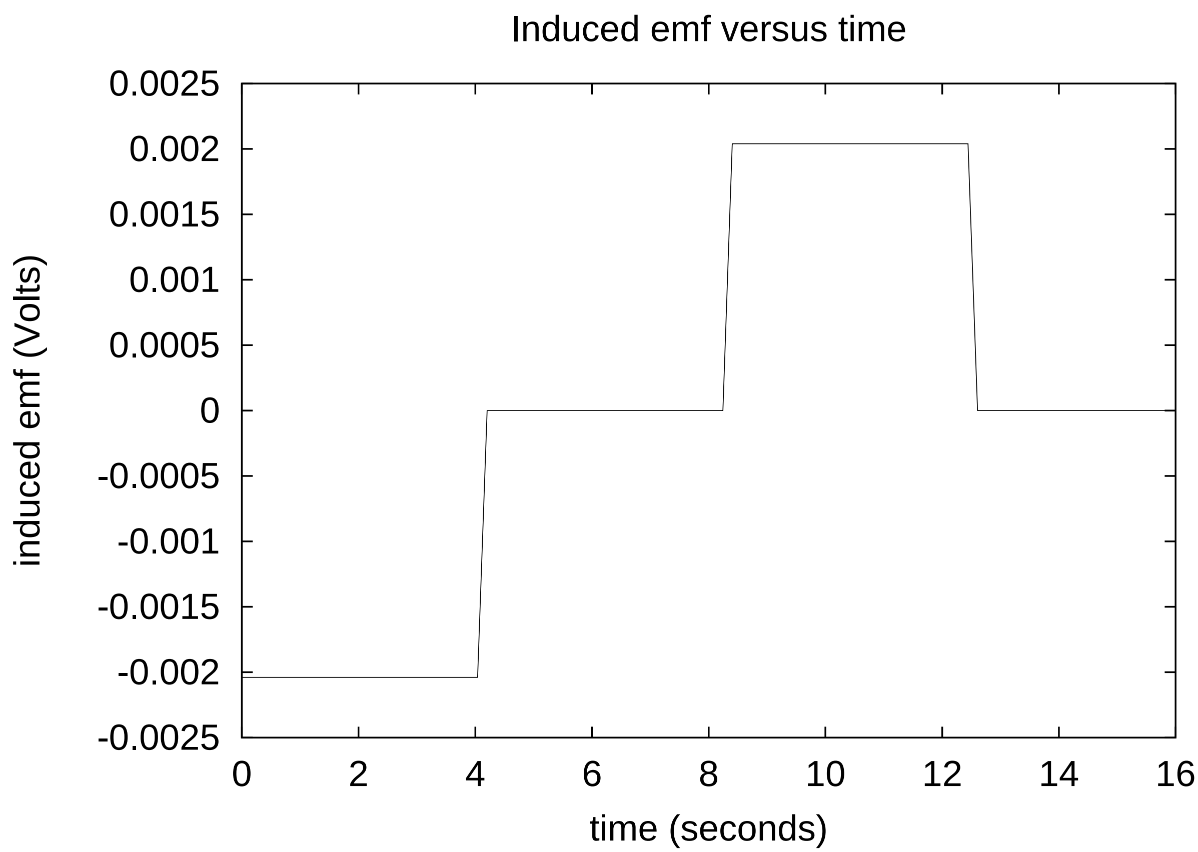
Faraday's law states the induced emf is

$$\varepsilon = -\frac{d\phi_m}{dt} \tag{12}$$

so, in each interval we have, from the above equations,

$$\begin{aligned} \varepsilon(t < t_1) &= -Bwv & (13) \\ \varepsilon(t_1 < t < t_2) &= 0 & (14) \\ \varepsilon(t_2 < t < t_3) &= +Bwv & (15) \\ \varepsilon(t > t_3) &= 0. & (16) \end{aligned}$$

So the graph for induced emf looks like...



I won't bother with the current because it just comes from $I = \varepsilon/R$, so it's trivial.

But notice the direction of the induced current. From Lenz's law it always flows such as to set up a magnetic field which opposes the *change* in the magnetic flux (it tries to maintain the flux).

For $t < t_1$ it wants to keep a zero magnetic flux so the induced current generates a magnetic field into the page. So, from the right-hand rule, the current must be clockwise.

For $t_2 < t < t_3$ (when the loop is leaving the magnetized area) the loop wants to keep it's strong magnetic flux so the induced current generates a magnetic field out of the page. So, from the right-hand rule, the current must be counter-clockwise.

8 Assigned Problem

(From Tipler Ch. 26 #70.)

A simple pendulum has a wire of length l supporting a metal ball of mass m. The wire has negligible mass and moves in a uniform horizontal magnetic field B. This pendulum executes simple harmonic motion with angular amplitude θ_0, in a plane perpendicular to $\mathbf{B}$. What is the emf generated along the wire?