Physics 153 Section T0H - Solution to Problem 7

Rik Blok

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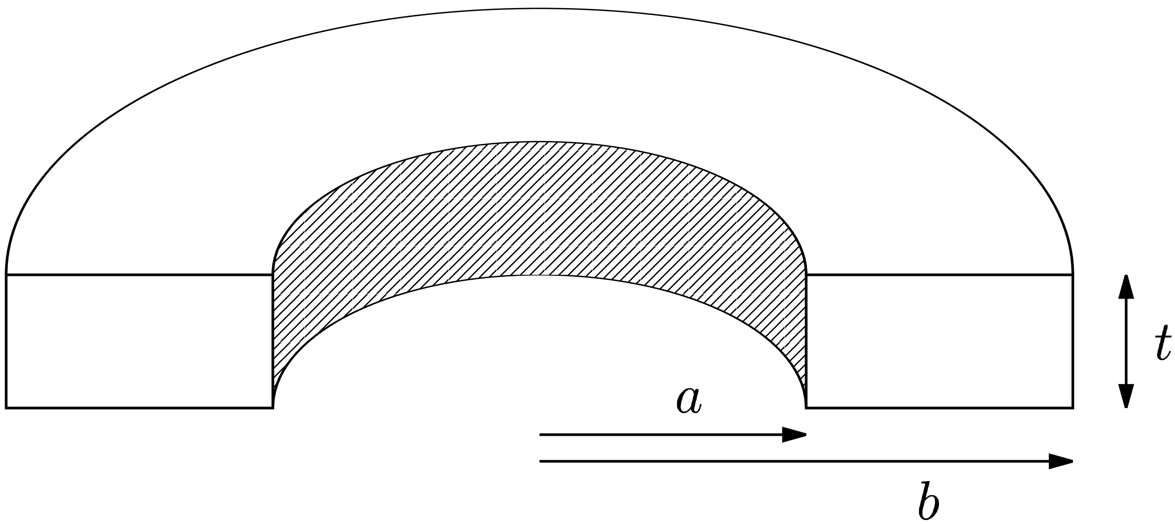
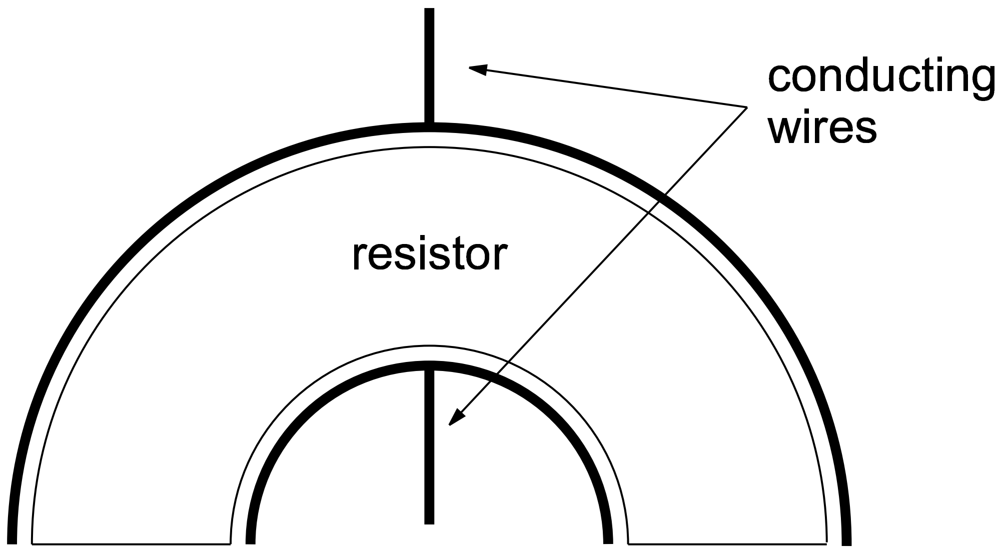
1 Assigned Problem



Find the resistance between the inner (shaded) and outer surfaces of the half ring shown above. The resistivity of the ring is ρ.

2 Solution

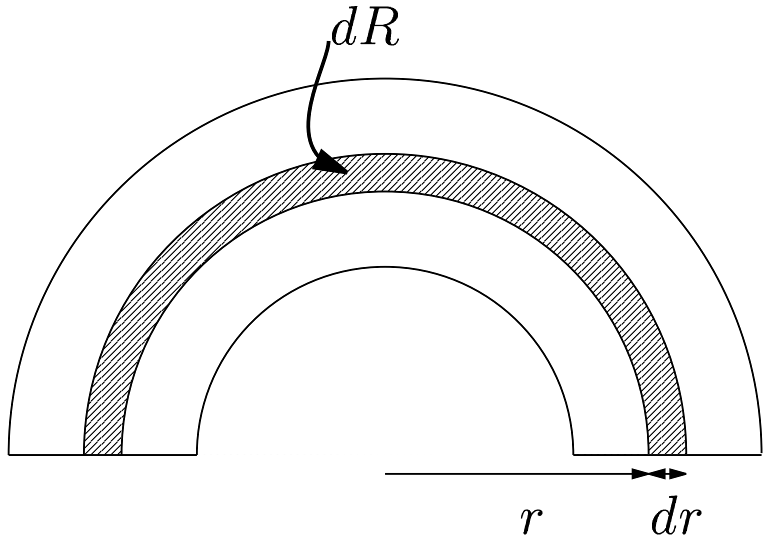


Schematically, we are trying to calculate the resistance of the resistor shown above.

We want to apply the equation

$$R = \rho \frac{L}{A} \tag{1}$$

but it doesn't apply to the whole object at once because the cross-sectional A (perpendicular to current) isn't a constant. But what we can do is divide the object into sections for which A *is* constant as follows:



Now A on the inner surface of the section is $\pi r t$ and on the outer surface of the section it is $\pi(r + dr)t$ so, if we take infinitesimal sections $dr \to 0$ then A is a constant for each section.

The length L (distance the electrons have to travel) of each section is just $L = dr$ so, for each infinitesimal section

$$dR = \rho \frac{dr}{\pi r t}. \tag{2}$$

Each of these resistors (one at every r-value) are lined up end-to-end (in series) so the equivalent resistance of all of them is just the sum (integral) over all of them

$$\begin{aligned} R &= \int dR \qquad (3) \\ &= \int_a^b \rho \frac{dr}{\pi r t} \qquad (4) \\ &= \frac{\rho}{\pi t} \int_a^b \frac{dr}{r} \qquad (5) \\ &= \frac{\rho}{\pi t} \ln(b/a). \qquad (6) \end{aligned}$$