Assignment 6 Solutions

6.2

(a) Solving $\hat{p} = \hat{p} - \mu\hat{p} - \nu\hat{p} + \nu$ for $\hat{p}$ yields $\hat{p} = \frac{\nu}{\mu+\nu}$.

(b) Either brute force iteration can be used, or we can transform the system by measuring the distance to the equilibrium. The latter approach is developed for this type of linear model in Recipe 6.1, where ρ is now $1 - \mu - \nu$. Either way the general solution is:

$$p[t] = (1 - \mu - \nu)^t p[0] + (1 - (1 - \mu - \nu)^t)\hat{p}$$

where $\hat{p} = \frac{\nu}{\mu+\nu}$.

(c) The general solution can be rearranged as:

$$p[t] = (1 - \mu - \nu)^t (p[0] - \hat{p}) + \hat{p}$$

which makes it easier to see that the distance from the equilibrium shrinks over time by an amount $(1 - \mu - \nu)^t$. Since the mutation rates μ and ν are both small, $(1 - \mu - \nu)^t$ will remain close to 1 for a long time, and thus the allele frequency approaches the equilibrium only very slowly. For example, if $\mu = \nu = 10^{-6}$, it would take 346,573 generations for the distance to the equilibrium to halve ($(p[t] - \hat{p}) = (p[0] - \hat{p})/2$ when $(1 - \mu - \nu)^t = 1/2$). [Answer must imply a slow approach to equilibrium with rare mutation.]