ANSWERS TO ASSIGNMENT # 6

1a) $\hat{p} = 15/174 = 0.0862$.

b) $s_{\hat{p}} = \sqrt{\dfrac{\hat{p}\hat{q}}{n-1}} = 0.0213$

c) $L_1 = \hat{p} - (1.96 s_{\hat{p}} + 1/(2n)) = 0.0415$

$L_1 = \hat{p} + (1.96 s_{\hat{p}} + 1/(2n)) = 0.1309$

$0.0415 \leq p \leq 0.1309$

d) Central Limit Theorem

e) The sum of a large series of random variables (or their average) has an approximately normal distribution.

f) Caution is warranted because the confidence limits obtained using the normal approximation may not be accurate when $\hat{p} < 0.1$ and $n = 174$ (Zar recommends a larger n in this circumstance).

2) $\mathrm{H_O}$: Rats do not prefer the balanced diet ($p \leq 0.5$)

$\mathrm{H_A}$: Rats prefer the balanced diet ($p > 0.5$)

Note: I decided that a one-tailed test is most appropriate if only two scenarios are likely: 1) rats cannot detect the balanced diet, in which case they should prefer the two foods equally; 2) they can detect the balanced diet, in which case they should prefer it. The χ^2 goodness-of-fit is a two-tailed test, and so the binomial test (using the normal approximation) is most appropriate here.

Under $\mathrm{H_O}$, $\mu = np = (72)(0.5) = 36$, $\sigma^2 = npq = (72)(0.5)(0.5) = 18$

$$P = P(X \geq 44) \approx P(Z \geq \frac{44 - 36 - \frac{1}{2}}{\sqrt{18}}) = P(Z \geq 1.77) = 0.0384$$

since $P \leq 0.05$, reject $\mathrm{H_O}$. Rats do prefer the balanced diet.

3a) $\mathrm{H_O}$: Survival rates of the two populations do not differ on acidic soils.

$\mathrm{H_A}$: Survival rates of the two populations differ on acidic soils.

	Dorval	Gaspé	row sums
Survived	41 (34.88)	27 (33.12)	68
Did not survive	58 (64.12)	67 (60.88)	125
column sums	99	94	193

$\chi^2 = 2.870$ ($G = 2.884$), $df = 1$ (use Yates correction)

$\chi^2_{0.05,1} = 3.841$. Since 2.870 not $> \chi^2_{0.05,1}$, $P > 0.05$, therefore do not reject $\mathrm{H_O}$

b) No, because there is always the probability of a Type II error.

4) $\mathrm{H_O}$: Disabling X_{ist} transcription does not change mean RNA levels ($\mu = 50$).
$\mathrm{H_A}$: Disabling X_{ist} transcription reduces or increases mean RNA levels ($\mu \neq 50$).

Assume that relative RNA expression in tissue cultures has a normal distribution in the population.
$\bar{X} = 69.16, \;\; s_{\bar{X}} = 12.892/\sqrt{5} = 5.766$
$t = \dfrac{69.16 - 50}{5.766} = 3.3231$
$t_{0.05(2),4} = 2.776$
Since $t > 2.776$, $P < 0.05$, reject $\mathrm{H_O}$.
Note: Since it is reasonable to expect that disabling X_{ist} would increase gene expression, a one-tailed test might be justifiable here.

5) The 2×10 measurements are not independent because measurements from the same individual are likely to be more similar to one another than measurements taken from different individuals. Hence they cannot be considered a random sample. However, the 10 salmon were randomly sampled and are therefore independent. A reasonable approach is to average the two measurements per individual and proceed with these $n = 10$ observations. The 10 averages are 146, 151, 154.5, 143, 154, 159.5, 157, 152, 147.5, 153.

Assume that blood sodium level of individual salmon has a normal distribution in the population.
$\bar{X} = 151.75, \;\; s = 5.057, \;\; s_{\bar{X}} = 5.057/\sqrt{(10)} = 1.599$
$L_2 = \bar{X} + t_{0.05(2),9} s_{\bar{X}} = 151.75 + (2.262)(1.599) = 155.37$
$L_1 = \bar{X} - t_{0.05(2),9} s_{\bar{X}} = 151.75 - (2.262)(1.599) = 148.13$
$148.13 \leq \mu \leq 155.37$